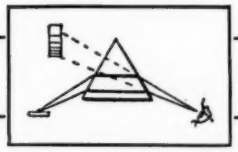
Journal of the OPTICAL SOCIETY *of* AMERICA

VOLUME 26
No. 6



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Published monthly
at Prince and Lemon Sts., Lancaster, Pa.
by the
AMERICAN INSTITUTE OF PHYSICS
Incorporated
175 Fifth Avenue, New York, N. Y.
FOR AND UNDER THE EDITORIAL SUPERVISION OF
THE OPTICAL SOCIETY OF AMERICA

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Journal of the
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VOLUME 26 JUNE, 1936 NUMBER 6

The Polarizing Characteristics of Polaroid Plates for Wave-Lengths 4000A to 20,000A

L. R. INGERSOLL, J. G. WINANS AND E. H. KRAUSE, *University of Wisconsin*
(Received March 30, 1936)

The polarizing property of the new Polaroid plates is found to be limited almost entirely to the visible spectrum, dropping off very rapidly in the infrared and violet regions.

THE probable increase in the use of the recently developed polarizing films or plates makes it desirable to determine their characteristics over a wide wave-length range. It is generally understood that these films are made by embedding minute dichroic crystals, such as those of herapathite, in a cellulose acetate film, aligning them by a stretching process and mounting between glass plates. Two plates, obtained from the Polaroid Corporation of Boston were tested. The percent of transmitted light which was polarized was measured for wave-lengths from 0.4μ to 0.6μ and at 0.8μ photographically, and from 0.7μ to 2.0μ with a polarizing spectrometer, using a thermocouple.[1]

For the photographic measurements, parallel light from a band filament lamp was passed through the Polaroid plate and then through two double-image prisms, and was then focused on the slit of a small Hilger spectrograph (E-31), using only one of the four images. Calibration exposures were made with the axes of the prisms at different angles, the Polaroid axis being parallel to one. Further intensity changes were obtained by changing the slit width. All exposures were the same (15 seconds). To determine the percent polarization, spectrograms were taken with the double-image prisms parallel and with the Polaroid plate parallel and crossed with them. Photographic plates were brushed continuously during development and intensities were determined from a Moll microphotometer record. Wratten M plates were used for wave-lengths less than 0.6μ and Eastman I-R plates for 0.8μ.

In using the infrared monochromator, a piece of heat absorbing glass was inserted in the light

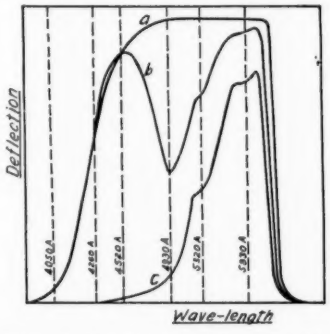

FIG. 1. Microphotometer records. *a* Polaroid parallel to calcite axes, *b* Polaroid perpendicular to calcite axes and with the intensity of light 8 times that for *a*. *c* Polaroid parallel to calcite axes with intensity of light 1/16 times that for *a*.

[1] For a description of this spectrometer see L. R. Ingersoll and W. R. Winch, Phys. Rev. **44**, 399 (1933).

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path for the measurements at 0.7μ and 0.8μ to minimize the effect of scattered long wave-length radiation.

A trace of a sample microphotometer record is shown in Fig. 1. The spectrum of the tungsten filament through the Polaroid shows distinct minima at 5320A and 5930A (Fig. 1, *b* and *c*). These are not accompanied by any noticeable change in the percent polarization for these wave-lengths. The minimum of the curve *b*, for the light through the Polaroid when crossed with the calcite, is a combination effect due to the change of radiation and polarizing effect with wave-length. The wave-length at the minimum is not that for maximum polarization.

The percent polarization is expressed in the usual way as the ratio of the differences of the two intensities in the crossed and parallel orientations of the plate and calcite to the sum of the intensities. The values obtained are plotted in Fig. 2. The curves show that the polarizing properties of these plates are practically limited to the visible spectrum. A Nicol or double-image prism, on the other hand, gives almost complete polarization over its entire range of transparency, or to beyond 2μ. A mica or Cellophane quarter-wave plate (for sodium light) showed about 1/6 wave retardation at 1μ, 1/9 at 1.5μ, and 1/12 at 2μ.

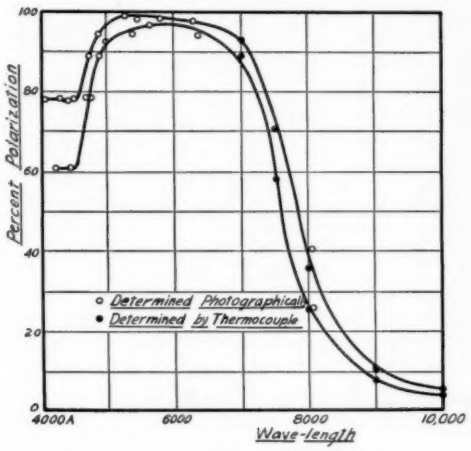

FIG. 2. Percent polarization for two samples of Polaroid plates. The polarization at 1.5μ (15,000A) is about 1 percent for each plate and at 2.0μ, it is 0.5 percent.

The general transparency of the Polaroid plates was also measured. Both plates were practically the same in this respect and showed, for unpolarized light, a transmission of 6 percent, at 4046A, 18 percent at 4358, 32 percent at 5460, 62 percent at 0.8μ, and about 80 percent beyond 1μ. The corresponding figures for light polarized in the plane of the Polaroid are 10, 31, 63, and about 82 percent at 0.8μ and beyond.

A Quantitative Study of the Improvement of Speculum Gratings by the Application of Aluminum

ALFRED B. FOCKE, *Brown University*

(Received March 6, 1936)

Tests made on a clean concave speculum metal grating show that if the surface is coated with aluminum by evaporation, the time required to obtain a photograph of a spectrum is reduced by a factor which is the ratio of the reflection coefficient of aluminum to that of the speculum metal. The speed of the original grating was increased for the given wave-lengths by the following amounts: 4077A–45 percent, 3341A–80 percent, and 2536A–170 percent. In preliminary tests the observed increase in speed was very much greater than indicated by these values, since the initial tests were made before the grating had been sufficiently cleaned.

INTRODUCTION

IN several recent papers J. Strong[1–4] and R. W. Wood[5] have discussed certain anomalies that have appeared in gratings coated with various metals. In two of these papers[1, 4] mention was made of the degree to which aluminum improved the reflecting power of speculum gratings. As there seemed to be no direct correlation between the reported improvements and the known differences in the reflection coefficients of speculum metal and aluminum, a report of the following experiment, which was carried through entirely without knowledge of Strong's measurements, seems justified.

The improvement of astronomical mirrors for work in the ultraviolet by the use of aluminum in place of silver suggested the study of the possibility of coating a speculum metal grating with aluminum, to determine whether or not it might be possible to increase its effectiveness in the ultraviolet region. It seemed probable that an aluminum-surfaced grating should be much faster than one wholly of speculum metal, since the reflection coefficient of speculum drops from about 60 percent at 4000A to 30 percent at 2500A, while that of aluminum falls from 87 percent to 80 percent in the same range.

The investigation was made on a Rowland concave grating of six-foot radius and 14,436 lines to the inch. It had been in regular laboratory use and had been carefully cleaned with precipitated chalk within the past year.

The evaporation apparatus used was very similar to the type developed by Strong[1] at the California Institute of Technology. It consisted of six tungsten filaments mounted in a ring twelve inches in diameter. The filament leads were brought through the metal bell jar by making use of ordinary spark plugs. These provided a simple means of heating the filaments separately so that a very uniform film of aluminum could be deposited in a vacuum upon any nearly plane surface that was twelve inches or less in its greatest dimension. The principal deviation from Strong's procedure was in the method of cleaning the grating to be aluminized. Instead of the potassium hydroxide solution which he used, a cold concentrated solution of sodium hydroxide was employed as a cleaning agent, since it is somewhat less active chemically. This did not harm the speculum and still was very effective in removing aluminum, thus making it possible to make measurements with or without aluminum at will.

A quartz mercury vapor arc was used as a light source for all spectra. A slit width of two millimeters permitted the use of a densitometer developed by C. W. Miller.[6]

RESULTS

In the first series of measurements a set of exposures was made, with the grating in just the condition in which it had been in laboratory use. The exposure times ranged from 30 seconds to 8 minutes. The grating was then aluminized and a similar set of exposures was made on the same

[1] J. Strong, Publ. A. S. P. **46**, 18 (1934).
[2] J. Strong, Phys. Rev. **46**, 326 (1934).
[3] J. Strong, Phys. Rev. **48**, 480 (1935).
[4] J. Strong, Phys. Rev. **49**, 291 (1936).
[5] R. W. Wood, Phys. Rev. **48**, 928 (1935).
[6] C. W. Miller, Rev. Sci. Inst. **6**, 125 (1935).

films. The startling result was found that the 30-second exposure taken with an aluminum surface was very much more satisfactory than the 8-minute exposure without aluminum. A rough estimate of the ratios of exposure times to obtain the same density varied from about 20 for the 4077A line to more than 100 for the 2536A line.

This result was so striking and so unexpected that the aluminum was removed from the grating with sodium hydroxide and a new set of exposures made. This time the great difference between the speculum surface and the aluminum surface disappeared. There was still a noticeable difference but it was of such an order that it required a more careful study.

The difference in the intensity of the spectra before aluminizing and after the aluminum had been removed was probably to be explained by the undoubted cleaning effect of the violent chemical reaction between the aluminum and the sodium hydroxide. The reaction would take place in the rulings as well as on the surface, thus freeing the grating of much dirt that could not be removed by any cleaning not involving such intimate turbulent action.

In the final series of measurements, a group of exposures was made which ranged from 1 second to 256 seconds. The times for consecutive exposures were increased by a factor of two, giving nine exposures for the plain speculum grating and a similar number for the grating when coated with aluminum. All films were given the same development treatment in order to reduce error that might enter from this cause. Characteristic curves were then plotted in order to correlate the observed line densities with the exposure times. Once these characteristics were obtained, a direct comparison of the exposure times could be made.

The result of such a comparison showed that the speed of the grating had been increased by the factor giving the ratio of the reflection coefficient of aluminum to that of speculum metal. This means that for the 4077A line the aluminum increased the speed by 45 percent, for the 3341A line by 80 percent, and for the 2536A line by 170 percent.

The difference between these results and those reported by Strong is probably due to a difference in the condition of the speculum surface, the useful measurements on speculum in this experiment being made immediately after the aluminum was removed.

Some typical spectra are reproduced in Fig. 1. The first is the spectrum obtained before the first aluminization with an exposure time of 8 minutes. The second is the one obtained after the first aluminization with an exposure time of 30 seconds. The great improvement is very noticeable. The third was taken after the aluminum had been removed, the exposure being 16 seconds, while the fourth is the 8-second exposure after the second aluminization.

The intensity of the 2536A line was measured in each of the first six orders with and without an aluminum surface. It was found that the ratios of the intensities in different orders were unchanged by the aluminum film. This, coupled with the observation that the ratio of line intensity to scattered light intensity was also unchanged, tends to indicate that there was no appreciable alteration in the shape of the rulings. A comparison of the densities of the spectral lines obtained after the surface had been coated with aluminum for the first time, with those obtained after it had been coated for the eighth time showed that there had been no change. This is conclusive evidence that the grating had not been harmed by the repeated application and removal of the aluminum.

A search was made for the presence of the Rayleigh bands discussed by Strong and Wood, but none were found. This would be in accordance with the findings of Strong,[4] as he specifically stated that they are not found in ordinary gratings, that is, gratings in which the width of the ruling is approximately one-half the grating space.

A consideration of this fact suggests that the intensity of the bands depends upon the width of the ruling. Consequently one would expect that the application of any metal to a grating which already showed bands, would cause an increase in their intensity, since some of the metal would be deposited upon the sides of the rulings. The relative amount that the ruling width would be decreased would be a function of the original width. A much greater change would be produced in the characteristics of a grating

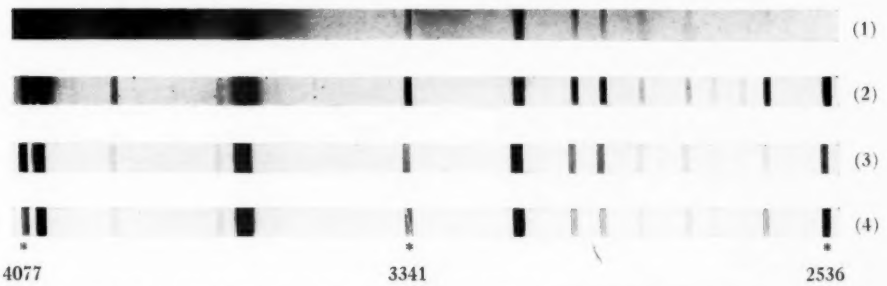

FIG. 1. (1) Before aluminization, 8-min. exposure; (2) after first aluminization, 30-sec. exposure; (3) aluminum removed, 16-sec. exposure; (4) after second aluminization, 8-sec. exposure.

possessing narrow rulings than for one of wide rulings.

It would also be expected that the effect of the applied film would depend upon the geometry of the evaporating apparatus, because the amount of metal deposited on the sides of the rulings would depend upon the angle of incidence of the impinging atoms. This idea may be carried further to offer a possible simple explanation for the different effects of different metals without the need of any change in the theory of the diffraction grating. To quote Strong[4] "Of the films tested, magnesium gave the strongest and most numerous bands with silver, aluminum, copper, and gold giving the effect in decreasing amount in the order given."

This is just the order of the ease of evaporation of the metals, with the possible exception of silver. Its boiling point is 150°C higher than that of aluminum, but it is well known for the ease with which it vaporizes from the solid state. The probability that any given atom should be reflected from a cold surface rather than condensing immediately upon it would certainly be greater, the smaller the difference between the temperature of that surface and the temperature at which normal evaporation could take place. This would indicate that the probability of reflection from the walls of the evaporating chamber and the consequent increase in the number of atoms striking the grating at large angles of incidence would be increased for the different metals in just the order given.

It would be an easy matter to test the correctness of this suggestion either by varying the geometrical arrangement of the evaporation apparatus or by changing the temperature of the evaporating filament. For a given metal, the lower the temperature at which the evaporation takes place, the lower would be the probability of reflected atoms reaching the grating surface. The writer was unable to make such tests as there was no grating available which had any indication of the bands.

The results of this investigation indicate the desirability of coating the surface of speculum gratings with aluminum in order to increase the intensity of the spectra produced by them. The increase in intensity obtained may be attributed both to the very effective cleaning that the grating receives as well as to the greater reflecting power of the aluminum surface. In the case of gratings with relatively wide rulings, no trouble with Rayleigh bands need be expected. Gratings with narrow rulings will probably develop more intense bands, but it may be possible to reduce the degree to which these bands appear by controlled conditions of evaporation.

The Optical Constants of Potassium

HERBERT E. IVES AND H. B. BRIGGS, *Bell Telephone Laboratories, New York, N. Y.*

(Received March 20, 1936)

INTRODUCTION

THE importance of a knowledge of the optical constants of the alkali metals is emphasized by numerous recent theories of the metallic state and the optical properties of metals in general. In these theoretical treatments the alkali metals, because of their extraordinary properties, in particular their spectral region of transparency, have figured largely. There has however existed a serious gap in our experimental knowledge, in that optical constants have been entirely lacking for the region of extreme interest, namely, the ultraviolet. Without such knowledge theories must stand unchecked. Sufficient warrant for undertaking an experimental determination of the optical constants of the alkali metals, of which this study of potassium is the first, is therefore found on this ground alone. In addition, the writers have a special interest in these constants in connection with their work on the photoelectric effect. A theory of the photoelectric emission from thin films of alkali metals, proposed some years ago,[1] which has been very successful in explaining the phenomena in the visible region of the spectrum, has urgently demanded optical data for its test in the ultraviolet region, where the most extreme and characteristic peculiarities of photoelectric emission are found.

METHOD

The method used is that described by Voigt[2] and used by Minor,[3] Meier,[4] Fréedericksz[5] and Erochin.[6] Later Weld[7] constructed a "crystelliptometer" following the method of Voigt. The theory of the method is given by Voigt and Minor.

In brief the method utilizes in addition to the usual Babinet compensator between crossed Nicols, a pair of quartz wedges, one of right, the other of left rotary quartz, mounted to form a plane parallel plate. In our work this plate was placed between the Babinet and the analyzer, its wedge edge perpendicular to the wedge edge of the Babinet. We will consider the incident beam to be plane polarized with the plane of vibration of the electric vector making an angle of 45° with the plane of incidence. Plane polarized light from the Babinet on passing through the rotary quartz suffers a progressive rotation of the plane of vibration, the amount depending on the difference in thickness of the two wedges. As viewed through the analyzer, the field shows a double rectangular array of dark spots, one array consisting of those points where the plane polarized light suffers a rotation of 0, $\pm\pi$, $\pm 2\pi$, etc., the other of those points suffering rotations

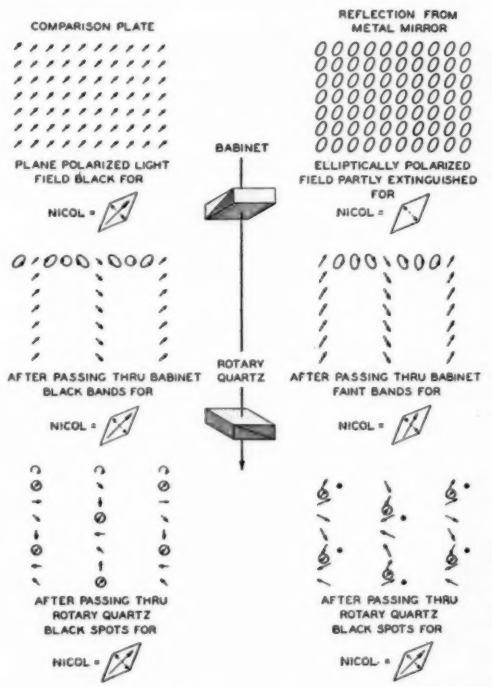

FIG. 1. Diagram of the formation of spot patterns for plane and elliptically polarized light.

[1] Herbert E. Ives, Phys. Rev. **38**, 1209 (1931).
[2] W. Voigt, Physik. Zeits. **2**, 303 (1901).
[3] R. S. Minor, Ann. d. Physik **10**, 581 (1903).
[4] W. Meier, Ann. d. Physik **31**, 1017 (1910).
[5] V. Fréedericksz, Ann. d. Physik **34**, 780 (1911).
[6] P. Erochin, Ann. d. Physik **39**, 213 (1912).
[7] Le Roy D. Weld, J. O. S. A. **6**, 67 (1922).

of $\pm\pi/2$, $\pm 3\pi/2$, etc. If the phase of the incident light is changed both arrays are shifted in the same direction by equal amounts. If the azimuth of the incident light is changed one array is shifted up, the other down. This behavior is illustrated in the two sets of diagrams of Fig. 1. The lower diagram at the left represents the null positions of the spots, obtained for light passing successively through the Babinet and rotary quartz. The corresponding diagram on the right represents the spot pattern for light reflected from a metal surface. The small black circles on this diagram are the null positions of the preceding diagram, inserted for comparison purposes.

The experimental procedure consists in general of determining the positions of these spots, either by visual measurement, or by photography, depending on the part of the spectrum under observation, and then calculating the refractive index and extinction coefficients, substantially by the method described by Minor.

Apparatus

In design our apparatus was made to follow as closely as possible that described by Minor. Instrument and optical parts were made by the Bausch and Lomb Optical Company.

In Fig. 2, S_1S_2 represents a quartz double monochromator of the Von Cittert type. The specimen and the various polarizing devices with the camera were mounted on a massive spectrometer-like instrument. The collimator aperture S_2 was circular; of diameter 0.8 mm. The polarizer P was of the Glan type, having a circular aperture of about 17 mm. The polarizer mount was equipped with a divided circle and vernier reading to 0.1°. The Babinet compensator B consisted of a fixed and a movable wedge each of an angle of about 7′. The wedges were mounted separately so that each was capable of slight rotation about the axis of the light beam for adjustment purposes. The movable wedge of the Babinet was provided with a micrometer screw reading to 0.01 mm. This came in use in the visible range where the spot pattern was too extended for the photographic method. The Babinet was mounted with its wedge edge perpendicular to the axis of the

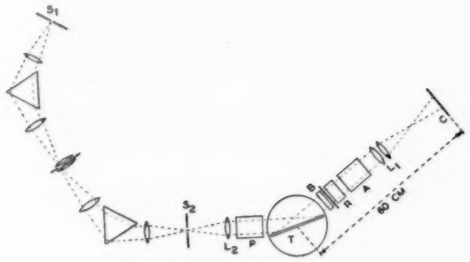

Fig. 2. The optical system.

instrument. The rotary quartz wedges R were of an angle of about 23°. The pair were mounted together and were capable of slight rotation about the axis of the beam for adjustment purposes. The cross hairs and a fiduciary mark were carried on a holder mounted midway between B and R. The analyzer A was a Glan-Thompson prism mounted with a divided circle and vernier reading to 0.1°. The lens pair L_1 was of crystal quartz calculated for minimum spherical aberration. C represents the position of the photographic plateholder. The effective aperture of the optical system was limited by the analyzer, whose dimensions were 15×20 mm. The camera L_1C was movable along the arm BC. The table T was similar to the ordinary prism table of a spectrometer, being provided with three leveling screws and capable of rotation about the axis of the instrument.

An aperture of diameter 0.7 mm, placed at the principal focus of L_1, was found to be of great help in suppressing stray illumination, caused chiefly by internal reflections at the various prism faces. The elements B to C were mounted on a rigid arm with a counterbalance, the arm rotating about the axis of the instrument. A scale on the base of the instrument, divided in degrees, gave the position of the arm and thus the incidence angle.

An auxiliary telescope with a Gauss eyepiece and leveling screw was clamped at the side of the camera. It came in use principally in adjusting the mirror surfaces on the prism table.

Adjustment of Apparatus

The adjustment of the several prisms was accomplished in the following manner: With the

FIG. 3. Spot patterns; comparison and reflection negatives for λ3650.

aid of a glass prism mounted on the table T the collimator and auxiliary telescope were made perpendicular to the axis of the instrument. The Glan polarizer was then adjusted so that the image of the slit as viewed through the telescope showed minimum shift on rotation of the Glan. The movement of the image could not be eliminated completely. The polarizer was then set for zero azimuth using light reflected from a prism face at the Brewsterian angle. The Glan-Thompson analyzer was also adjusted for minimum shift on rotation and set for extinction with the polarizer. The Babinet-rotary quartz mount, with the prisms removed, was put in place and the cross hairs made parallel and perpendicular to the axis of the instrument. One wedge of the Babinet was put in position and adjusted until the field was black. The other wedge was adjusted with respect to the first until the field was again black. With a Gauss eyepiece in place of the collimator slit the two Babinet wedges were brought parallel to each other and perpendicular to the axis of the collimator. The rotary quartz prism was then mounted and brought into parallelism with the Babinet. The rotary quartz unit was then rotated until its bands were parallel to the vertical cross hair. This final adjustment was completed from measurements of photographic negatives.

METHOD OF MAKING OBSERVATIONS

A typical set of exposures for a particular wave-length consisted of four, two comparison or "null position" exposures and two for light reflected from the metal. For the comparison exposures the camera arm BC was made parallel to the collimator S_2L_2. Two exposures were made at this camera setting, one for the polarizer at 45° azimuth, the other with the polarizer at 135°

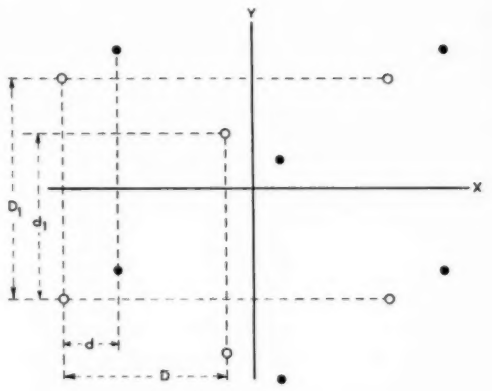

FIG. 4. Relation between comparison and reflection patterns.

azimuth. The metallic mirror was then adjusted on the leveling table T with the aid of the auxiliary telescope clamped at the side of the camera. With the aid of the Gauss eyepiece the mirror surface was made perpendicular to the axis of the collimator. It was then set for the particular incidence angle desired, the camera brought into position and two exposures to correspond with the two comparison exposures taken. Eastman Process plates were found suitable for most of the work, although at λ2536 better results were obtained with Eastman 50 plates. Exposure time varied from 15 minutes at λ3650 to 4 to 6 hours at λ2536.

In Figs. 3 and 5 are shown the appearance of several negatives. (In the original negatives the fine images of the cross hairs are clearly visible, but are hard to trace in the reproductions.)

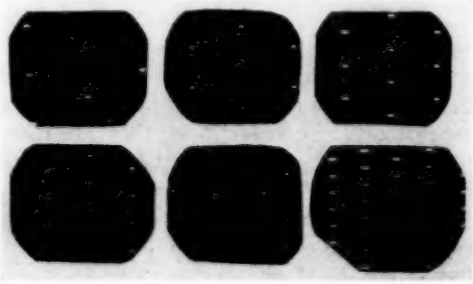

FIG. 5. Comparison plates, showing variation of spot spacing with wave-length. Top row, left to right, λ4358, 4047, 3650. Bottom row, λ3341, 3126, 2536.

Notation Employed

Before describing the method of measurement of the plates, and the method of utilizing these measurements, it is desirable to make a statement on the notation used by us for the various quantities and derived quantities used, since there is considerable confusion on this subject in the literature.

The notation adopted here is that used by Fry.[8] In this notation ϕ is the relative phase shift under the assumption that for increasing incidence angle ϕ decreases from 180° to 0°. The azimuth of restored linear polarization is designated by θ. In accord with modern usage, which attempts to avoid the term "plane of polarization," this is the angle between the plane of *vibration of the electric vector* and the plane of incidence. In the notation used by Drude the azimuth of restored linear polarization was measured by the angle between the "plane of polarization" and the plane of incidence and was designated by ψ. The relation between the two is obviously $\theta=90-\psi$.

Measurement of Plates

The plates were measured with the aid of a Zeiss toolmaker's microscope provided with a two-way stage such that the x and y coordinates of the spot centers could be obtained with a single setting. Settings of the two screw drums were read directly to 0.01 mm, fractional hundredths being estimated. In practice the plate was secured to the microscope table by means of a clamp. It was then oriented until the cross hairs on the plate coincided with those of the microscope eyepiece. This was readily done as the table could be rotated in a horizontal plane. The coordinates of the intersection of the cross hairs were then noted, after which the positions of the spot centers and the end of the fiduciary mark were recorded. The distance from the fiduciary mark to the intersection of the cross hairs was a measure of the magnification. In practice no correction for magnification was found to be necessary.

The determination of the phase shift ϕ and azimuth of restored linear polarization θ was readily made in terms of the x and y coordinates of the spot centers. With reference to Fig. 4, which is an enlarged copy of the final diagram of Fig. 1, let the open circles represent the spot positions of the reflection plate, the small closed circles those of the comparison or null position plate. As compared with their null positions, it is to be noted that all spots of the reflection plate are shifted an equal amount in the x direction, while in alternate columns the spots are shifted up or down. Let D represent the distance between consecutive columns and D_1 the distance between consecutive spots in the same column. In the one case the distance corresponds to a phase change of 180°, in the other to a rotation of the plane of vibration of 180°. The distance d is a measure of the phase shift obtained from a comparison of the average abscissae of spots of corresponding columns on comparison and reflection plates. The distance d_1 is a measure of the rotation of the plane of vibration, obtained from measurements on the reflection plate alone. The following relations, giving the ϕ and θ used in the computations, are easily verified:

$$\phi=(d/D)\times 180, \quad \theta=(d_1/D_1)\times 90.$$

The formula used for the calculation of the optical constants is that developed by Fry[9] (Eq. (55)).

$$N+iK_0=\tan I\left[\frac{(\rho+1)^2-4\rho\sin^2 I}{(\rho+1)^2}\right]^{\frac{1}{2}},$$

where $$\rho=e^{i\phi}\tan\theta.$$

The above relations hold for light incident through air or vacuum. It is still valid for light incident through a medium of refractive index N', provided $N+iK_0$ is replaced by $(N+iK_0)/N'$.

Errors

Errors in phase and azimuth determinations were reduced by the usual method of averaging the results obtained for light whose plane of

[8] T. C. Fry, *Plane Waves of Light, I, II and III*, J. O. S. A. **15**, 137 (1927); **16**, 1 (1928), 307 (1932).

[9] T. C. Fry, J. O. S. A. **16**, 1 (1928). Dr. Fry has asked us to call attention to an error, which crept into his paper in the process of changing over to a more logical system of notation than that in which it was originally written. The error first appeared in the unnumbered equation preceding (55), p. 23, where the factor $r/c(\rho+1)$ should have been $r/c(r+1)$ and was continued in Eq. (55), which should have been as given above. Beyond that point it does not again appear.

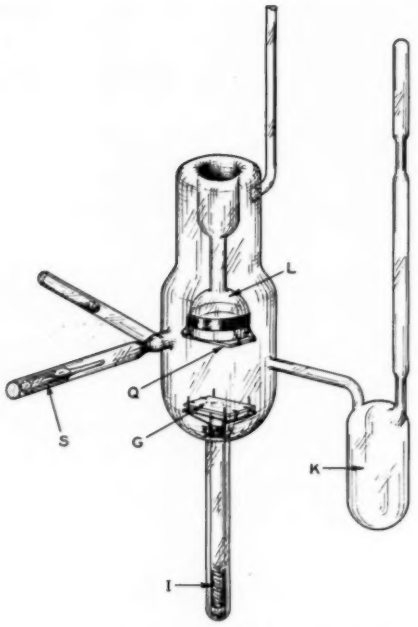

FIG. 6. Tube for preparing alkali metal surfaces.

vibration made, respectively, the angles 45° and 135° with the plane of incidence.

Oblique transmission of the light through the Babinet rotary quartz system results in a distorted pattern. The effect for the Babinet is more serious than for the rotary quartz. Errors arising from inexact adjustment of the mirror surface may be reduced by mounting the Babinet with its wedge edges parallel to the plane of incidence. In this position oblique incidence in the horizontal plane is less effective than in the vertical plane. The advantage of the above arrangement follows from the fact that adjustment of the mirror surface parallel to the axis of the instrument usually may be made more accurately than the setting for incidence angle. For the same reason it follows that parallax errors are reduced.

In the work which follows, the mirror surface was bounded by the base of a 60° quartz prism (Fig. 7). With the auxiliary telescope set perpendicular to the prism face coincidence of the telescope cross hairs with the center of the image of the collimator aperture was an accurate check for 60° incidence.

The Babinet and rotary quartz wedges used in this work were designed for the wave-length range λ2500 to λ4500. At 2536 the spot pattern consisted of four phase columns, with six to eight spots in each column. At 4358 the pattern was quite extended, usually consisting of two phase columns with two spots in each column. The several spots in a single phase column should lie in a straight line, i.e., the Babinet band. Random departures from the straight line were consistently noted, the magnitudes of which were decidedly greater than the measurement errors in locating the centers of the spots. In a test made at λ3650, the maximum variation from the mean for single spots was ±1.5°. Such effects are to be expected from variations in flatness of the Babinet surfaces.

According to the manufacturers, the surfaces of the Babinet were worked to λ/8. For the four surfaces the maximum deviation would thus be ±λ/4. At λ3650 the phase variations introduced by a variation in thickness of ±λ/4 is ±1.5°.

The variations above described are for single spots. Where the number of spots available for the determination of the position of a phase column is limited to two, as is the case in the visual region, the error may be large, i.e., ±1°. In the ultraviolet where the number of spots per phase column was greater, the errors from the above source were correspondingly diminished.

Errors arising from variations in flatness of the surfaces of the rotary quartz prisms were of negligible influence in the determination of azimuth. These determinations were very satisfactory, being of the order of accuracy of the setting of the polarizer, i.e., ±0.1°.

CHOICE OF METHOD FOR MAKING MEASUREMENTS ON POTASSIUM

As the preparation and measurement of a free mirror surface of potassium involves many

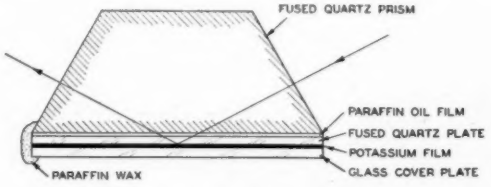

FIG. 7. Quartz prism with potassium mirror attached.

difficulties, such as the tendency of the surface to reticulate, the difficulty in obtaining strain free quartz windows, through which to observe the mirror when obtained, etc., it was decided to investigate a surface deposited on a quartz plate, making measurements through a quartz prism on the quartz-potassium surface. The reduction of the values so obtained to those for a free potassium surface was made by the usual assumption that the refractive index of the quartz-potassium surface must be multiplied by the refractive index of the quartz to get the air values, while the extinction coefficient K remains unchanged. Ample evidence for this is found in the work of Meier[4] and Erochin[6] on mercury, and Ingersoll and Birge[10] for several metals in contact with transparent media.

As the formula used in our computations gives N and $K_0 = NK$,[11] both N and K_0 must be multiplied by the refractive index of the quartz to get the air values.

Previous investigators who have used this method have worked with 45° glass prisms. We chose to work at 60° because our oblique incidence photoelectric measurements, to which we intended to apply our results, were made at this angle. Obtaining a fused quartz prism of the requisite size and quality was a matter of great difficulty, which was finally solved by the offer of Mr. Twyman, of Adam Hilger & Company to watch the large quantities of fused quartz passing through their hands, until a satisfactory piece was found. The quartz plates, which did not present the same degree of difficulty, were also from material selected by Hilger. Visual examination between crossed Nicols showed them nearly free from strains. As a check on the quartz prism alone, light was internally reflected at 60° incidence with the following results:

λ	I	ϕ	θ	N	$N(I.C.T.)$
3650	60°	39° 36′	45.0°	1.477	1.475

In order not to subject the quartz prism to unnecessary risk, the potassium surface was deposited on one of the strain-free fused quartz plates and backed with a microscope cover glass as described in a later paragraph. It was then sealed to the quartz prism with vacuum distilled paraffin oil. As a test to determine whether the introduction of the oil film and the quartz plate produced any serious effect, the following experiments were performed. A mercury trough was constructed with the quartz prism as one side. Light was reflected from the quartz mercury surface and the optical constants of the mercury determined. The trough was then removed, the quartz plate sealed in the side previously occupied by the quartz prism, and the cell sealed to the quartz prism with paraffin oil. Light was again reflected from the quartz-mercury surface, this time passing through the oil film and quartz plate in addition to the quartz prism. The results are given in Table I, the first set referring to the quartz prism-mercury surface, the second to the quartz plate-mercury surface.

TABLE I.

λ	I	ϕ	θ	N	K_0	K
5461	60°	134° 12′	51° 15′	1.53	4.65	3.04
5461	60°	135° 24′	51° 22′	1.63	4.76	2.93

The differences in values of ϕ and θ shown above are of about the order of accuracy of the measurements at that wave-length. Whether or not the introduction of the quartz plate and oil film produce a change in ϕ and θ, it is safe to assume that this change is not of serious amount.

For the purpose of comparison, values of N, K_0 and K, interpolated for λ5461 from several sources are given in Table II.

TABLE II.

	N	K_0	K	
Meier	1.45	4.17	2.86	Hg against UV-glass
Erochin	1.44	4.03	2.80	Hg free surface
Duncan and Duncan[12]	1.71	5.16	3.02	Hg against glass
O'Brien and Wheeler[13]	1.35	4.60	3.40	Hg free surface

The values given by Meier for Hg against UV-glass and by Erochin for the free surface were obtained using the same measuring apparatus. They are thus particularly suitable for a comparison of the constants for a free surface and

[10] L. R. Ingersoll and R. T. Birge, Phys. Rev. **29**, 392 (1909).

[11] For the relation between K and K_0 see T. C. Fry, J. O. S. A. **15**, 143, 144 (1927), footnotes.

[12] R. W. and R. C. Duncan, Phys. Rev. **1**, 294 (1913).

[13] Brian O'Brien, Phys. Rev. **27**, 93 (1926).

those for a surface against glass. Their values are in exceedingly good agreement for the range common to the two experiments, i.e., λ3255 ×λ5080. Our values for Hg against glass lie between the values given by Meier and those given by Duncan and Duncan. The group as a whole exhibits the decided variations which are characteristic of determinations of optical constants by katoptric methods, attributable in part to differences in surface conditions, in part to the extreme sensitiveness of the derived quantities N and K_0 to relatively small errors in the observed quantities due to the character of the formulae used.

Preparation of the Potassium Surfaces

The surface used was in the form of a sandwich of potassium between two plates, one of glass, the other of strain-free optically flat fused quartz. The method of preparation is best described by referring to Fig. 6. Q represents the quartz plate; L is a small liquid-air container.

Potassium from the bulb K was condensed on Q to form a heavy coating. The tube was then sealed off from the pumps, partially inverted, and the glass cover plate G was caused to fit over Q. A magnetic control I assisted in this operation. A gentle current of warm air was then directed into L causing the potassium to liquefy and bringing both plates into complete contact with the potassium. A small vial of vacuum distilled paraffin oil in the side tube S was then broken and the oil allowed to drip over the plates. The tube was then opened, the plates removed, and the excess potassium wiped off under oil. After removing the excess oil the mirror was sealed to the 60° quartz prism, paraffin oil being used, after which the mirror and prism were dipped in melted paraffin wax to hold the mirror firmly to the prism. Fig. 7 illustrates the general appearance of the mirror and prism ready for measurement.

Potassium surfaces prepared in this way were very good, but the method of sealing with paraffin oil and paraffin wax was not entirely satisfactory. The presence of the oil in contact with the wax ultimately caused a softening of the wax. Leaks developed, resulting in a slow deterioration of the potassium surface, beginning at the edges and gradually reducing the aperture. However such surfaces last long enough for a complete series of measurements.

Results of Measurements on Potassium

In Table III the values of ϕ and θ for the wavelengths 2536, 3126, 3650 and 4047 were obtained by the photographic method for which the instrument was designed. For values of λ greater than 4047 the spot pattern for the wedges used was too extended for the photographic method. The measurements at λ4358, 5461, and 5780 were made visually, by using the micrometer screw attached to one of the Babinet wedges for the determination of phase shift, the analyzer

Table III. *Optical constants of potassium.*

λ	I	ϕ	θ	N	K_0	R (calc)
5780	60°	85° 18′	46° 2′	0.094	1.57	89.8
5461	60°	82° 0′	46° 1′	0.091	1.42	88.7
4358	60°	72° 6′	46° 16′	0.121	0.978	78.1
4047	60°	66° 48′	46° 2′	0.105	0.710	73.2
3650	60°	62° 36′	45° 56′	0.150	0.443	60.7
3126	60°	57° 30′	45° 32′	0.410	0.080	18.2
2536	60°	50° 48′	45° 42′	0.744	0.049	2.61

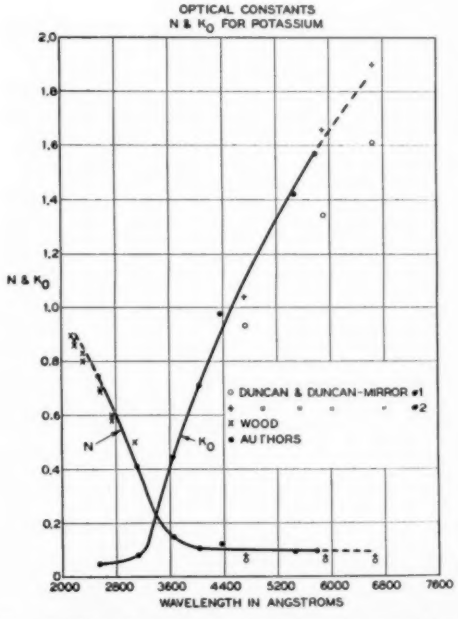

Fig. 8. Dispersion curves of N and K_0 for potassium.

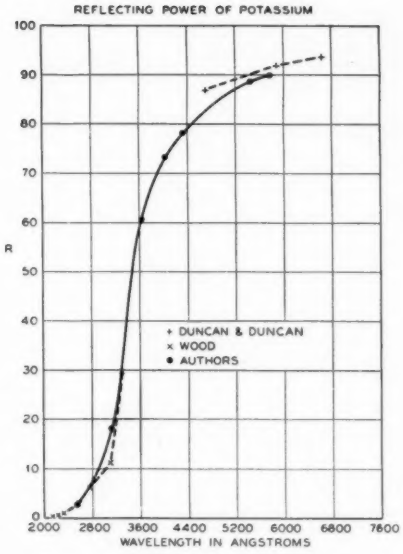

FIG. 9. Reflecting power of potassium.

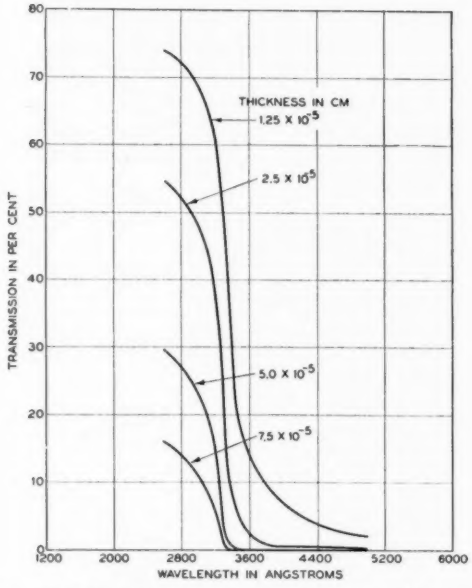

FIG. 10. Calculated transmissions for several thicknesses of potassium.

being utilized for the determination of azimuth of restored polarization.

In the course of the work three potassium mirrors were prepared by the method described above. Complete series of measurements were not obtained with the first two, because of deterioration of the surfaces or other defects. The data given in the table were all obtained from the third mirror. The number of plates taken and measured varied from four (at λ2536 and 3126) to eight (at λ3650). The visual measurements were at each wave-length the mean of eighty settings to determine phase, and a like number to determine azimuth.

Fig. 8 shows the values of N and K_0, and Fig. 9 the values of the reflecting power. Wood's[14] values and those of Duncan and Duncan are included for comparison purposes. For the range common to the two experiments our calculated reflecting powers agree very well with those observed directly by Wood, with the exception of our value at 3126 which is decidedly higher than the value given by Wood for 3100. For the visible range our values of N and K_0 agree very well with those obtained by Duncan and Duncan for their mirror No. 2.

Transmission curves calculated from interpolated values of K_0 are shown in Fig. 10. The calculated transmissions are in good agreement with the photographic results for varying thicknesses given by Wood. An abbreviated table of calculated transmissions is given (Table IV).

TABLE IV. *Transmission.*

Thickness (cm)	λ5000	λ4500	λ4000	λ3000
7.5×10^{-5}	1.0×10^{-10}	1.2×10^{-9}	1.1×10^{-7}	0.11
5.0×10^{-5}	2.2×10^{-7}	1.1×10^{-6}	2.3×10^{-5}	0.23
2.5×10^{-5}	4.7×10^{-4}	1.1×10^{-3}	4.8×10^{-3}	0.48
1.25×10^{-5}	0.02	0.03	0.07	0.69

For a film 5×10^{-5} cm thick the transmission at λ3000 is 23 percent. For this film the transmission in the blue is such that a bright light source would be faintly visible through the film. Wood states, "A film of potassium through which the sun's disk is barely visible (which means a reduction in intensity of half a million) transmits 25 percent of everything below 3000."

COMPARISON WITH THEORY

We select for comparison the optical constant values calculated by Kronig[15] on the basis of

[14] R. W. Wood, Phys. Rev. **44**, 353 (1933).

[15] R. de L. Kronig, Nature **133**, 211 (1934).

relations developed by him and by Zener.[16] The curve of the values so obtained (see Fig. 11) shows a general resemblance to our curve for experimentally determined values. By using a smaller value of δ (see Kronig's Eq. (2)) better agreement between the calculated and experimentally determined values was obtained for values of K_0 below the critical frequency and of N above the critical frequency. The extremely small values of N and K_0 predicted by the theory for the rest of the range are only approximately realized experimentally.

The value of $Ne^2/(2\pi m\sigma_0)$, used by Kronig in his calculation for δ, is given by the theory as the upper limit of δ.[17] For the infrared range Kronig compares the values of the quantity $Ne^2/(2\pi m\sigma_0)$ with values of δ calculated from the optical constants for several metals other than potassium, and finds that this value is consistently too great, in the case of copper by a factor of nearly three. In view of these facts, our empirical use of the value 2.9×10^{12}, instead of the value $Ne^2/(2\pi m\sigma_0)=4.11\times10^{12}$, seems to be not implausible, especially since it brings about substantial agreement between the experimental and theoretical curves in respect to the significant wave-length at which $e=0$, where the metal becomes transparent.

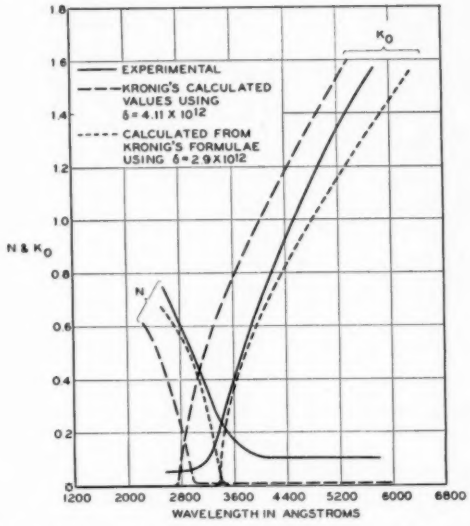

FIG. 11. Comparison of experimentally determined optical constants with those calculated from Kronig's theory.

[16] C. Zener, Nature **132**, 968 (1933).

[17] R. de L. Kronig, Proc. Roy. Soc. **A133**, 255 (1931). See relation (15). See also A. H. Wilson, Proc. Roy. Soc. **A151**, 282, 283, 284 (1935).

Calculated and Experimental Photoelectric Emission from Thin Films of Potassium

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(Received March 20, 1936)

INTRODUCTION

SEVERAL years ago one of the writers[1] proposed a theory of the photoelectric emission from thin films of alkali metals on supports of other metals, not photoelectrically active in the regions of the spectrum under observation. According to this theory the photoelectric emission is proportional to the rate of energy absorption by the thin film of alkali metal. The magnitude of the photoelectric current depends on the energy density immediately above the supporting metal, which is established from a knowledge of the optical constants of that metal, and upon the specific absorption of the alkali metal film. For its verification, the theory demands a knowledge of the optical constants of both supporting and alkali metals throughout the whole region of the spectrum where observations can be made. While optical constants have been determined for platinum, which is the metal most commonly used for a support for these thin films, no optical constants for the alkali metals have been available except in the visible region. In this region, a very satisfactory confirmation of the theory was obtained, particularly in respect to the variation of emission with the angle of incidence for the two principal planes of polarization (vectorial effect). One of the most characteristic phenomena of photoelectric emission from thin films, namely the occurrence of a pronounced maximum of emission in the spectrum, could not be compared with the predictions of this theory because these maxima in the case of the alkali metals lie in the ultraviolet. The theory of photoelectric emission from thin films has consequently had to stand unconfirmed in its entirety until such time as the optical constants of the alkali metals became available. In a separate paper,[2] the writers describe an experimental determination of the optical constants of potassium, which are here listed in Table I. In the present paper, these constants are applied to the photoelectric theory, and the results are compared with experiment.

TABLE I. *Optical constants of potassium.*

λ	N	K_0
5780	0.094	1.57
5461	0.091	1.42
4358	0.121	0.978
4047	0.105	0.710
3650	0.150	0.443
3126	0.410	0.080
2536	0.744	0.049

THEORY

For the case of very thin films of alkali metals as spontaneously deposited on a base, such as platinum, the effect of the alkali metal film on the light reflected from the base may be ignored. Under these conditions, the theory is summarized in the following three statements:

(1) The components of electric intensity that exist just above the surface are calculated as they would exist for the surface *between air and the supporting metal.*

(2) The normal component is corrected for the discontinuity which must take place on crossing the surface *between air and the alkali metal film.*

(3) The photoelectric current is proportional to the rate at which energy is absorbed by the film material from a wave of the amplitude as thus computed. The complete mathematical statement of these operations is given in the following expression:[3]

$$A_L \doteq \frac{\left(\frac{N_1 K_{10} p}{2\mu_1 \pi}\right)\left(\left|\frac{\mathfrak{E}_x}{E}\right|^2 + \left|\frac{\mathfrak{E}_y}{E}\right|^2 + \left|\frac{g}{g_1}\right|^2 \left|\frac{\mathfrak{E}_z}{E}\right|^2\right)\zeta}{(N\Lambda_0 p \cos I/8\pi^2\mu)}.$$

[1] Ives, "The Vectorial Photoelectric Effect," Phys. Rev. **38**, 1209 (1931).

[2] Ives and Briggs, "The Optical Constants of Potassium," J. O. S. A. this issue.

[3] The complete theory is given in the paper above cited (reference 1) and in mathematical detail by Fry in "Plane Waves of Light III" (J. O. S. A. **22**, 307 (1932)) from which the formula here given is quoted. This latter paper should be consulted for the meaning of the various symbols, and for the methods of computation.

In this expression, the refractive index N_1 and extinction coefficient K_{10} of the alkali metal appear both as multipliers in the numerator, and in the discontinuity expression

$$g/g_1(=N^2/(N_1+iK_{10})^2)$$

while the optical constants of the underlying metal occur in the terms $\mathfrak{E}_x$, $\mathfrak{E}_y$, $\mathfrak{E}_z$, taking forms appropriate to the angle of incidence and plane of polarization of the incident light. N, the refractive index of the medium above the irradiated plate (air) is, in our experiments, unity, $\Lambda_0 p/2\pi$ is the velocity of light in the same medium, consequently the denominator is a constant. ζ is the thickness of the film, an unknown constant.

Computations

We have chosen to present the predictions of the theory before giving experimental results, even though our experimental data, and those of other investigators, have been available for some time. This we do because we believe that the phenomena are more accurately indicated by the theory than by the results of the direct measurements, due to the intrinsic difficulties inherent in the latter. We have in our computations used both platinum and platinum-iridium as our underlying metals. This requires a word of explanation. We have consistently observed in our photoelectric measurements, in which obliquely incident light polarized with the plane of vibration parallel and perpendicular to the plane of incidence is used, that the ratio of emission for these two conditions is always greater the more nearly the surface approaches a completely specular character. This behavior is so marked as to invite the suspicion (considering only the experimental indications) that with a really perfect mirror there would be *no* emission for light with its plane of vibration parallel to the surface! Furthermore, in obtaining response curves (in the ultraviolet region) the curves for light vibrating parallel to the surface usually appear to be merely a much reduced edition (about 1/20) of the curves obtained for the other plane of polarization; indicating that such curves as were obtained were due principally to light depolarized and scattered by the windows of the cells and elsewhere in the optical path. Now platinum will not take a really high polish, whereas platinum-iridium (15 percent iridium) will take a very good polish, and was therefore preferred as a base in the experimental work.

The use of platinum-iridium in the experiments demanded a knowledge of its optical constants, and these we have obtained by the same apparatus and procedure as for the alkali metals. The values obtained are given herewith, in Table II.

Table II. *Optical constants of platinum-iridium (85 percent Pt, 15 percent Ir).*

λ	N	K_0
5780	2.12	3.41
5461	2.15	3.43
4358	1.67	2.68
4047	1.62	2.52
3650	1.64	2.37
3126	1.52	2.04
2536	1.36	1.76

The values of the optical constants of platinum were taken from Meier, and have been plotted in a previous paper.[1]

The computations of photoelectric emission were all made to apply to our uniform experimental conditions, in which light is incident on the surface at 60°, and the observations were made with light polarized in the two principal directions, designated as || (electric vector parallel to plane of incidence) and ⊥ (electric vector perpendicular to plane of incidence). The results of the computations for the expected emissions are given in Tables III and IV. In Figs. 1 and 2 are plotted the values for the || case (in relative values, since the thickness ζ is unknown). The values for the ⊥ case are too small to show on this plot. The outstanding predictions of the

Table III. *Absorption, thin film of potassium on platinum-iridium.* (Relative values.)

λ	$A\perp$	$A\parallel$
5780	0.233	2.37
5461	.249	2.58
4358	.441	9.23
4047	.332	19.3
3650	.348	97.6
*3500	.385	210.0
*3400	.406	273.6
*3300	.372	242.2
3126	.250	78.1
*2800	.311	20.27
2536	.429	9.86

* Optical constant values obtained by interpolation.

TABLE IV. *Absorption, thin film of potassium on platinum.* (Relative values.)

λ	$A\perp$	$A\parallel$
5780	0.232	2.01
5461	.229	2.53
4358	.340	9.96
4047	.255	21.88
3650	.360	100.9
*3500	.417	213.4
*3400	.444	275.5
*3300	.406	244.1
3126	.257	80.92
*2800	.310	20.99
2536	.523	8.88

* Optical constant values obtained by interpolation.

theory are, *first;* the occurrence of a very sharp maximum of emission at 3400A for light polarized with the electric vector parallel to the plane of incidence; *second*, an extremely small emission for light polarized with the electric vector parallel to the surface, which varies in amount by no more than a factor of two throughout the spectral range studied. *The ratio of the values of emission for the two planes of polarization at the maximum is approximately 600 to 1.*

COMPARISON WITH EXPERIMENT

Experimental data were obtained from photoelectric cells in which a thin platinum or platinum-iridium (85 percent Pt, 15 percent Ir) plate, capable of being heated to incandescence by electron bombardment, was allowed to become photosensitive by spontaneous deposition of potassium from the metal vapor coming from extended surfaces of potassium on the walls. These cells, and also the ultraviolet polarization monochromator, energy and current measuring apparatus, have been described in previous papers.[4] A typical experimental curve for a potassium film on platinum-iridium is shown in Fig. 1 along with the computed curve. The emission curve for light polarized with the electric vector parallel to the surface is omitted, since, as stated above, it is merely a scattered light curve of approximately five percent the value of the curve shown. We believe, on the basis of the computed results, that the true curve would be represented on this scale, by a line practically coincident with the axis. Referring now to the experimental curve for light polarized with the electric vector parallel to the plane of incidence (||), we note first of all, a very close agreement of the position of the maximum with the computed value. The experimental peak is shifted toward longer wave-lengths by a hundred Angstrom units, in this particular case, but by varying amounts in different experiments, as will be discussed shortly. The agreement in shape on the short wave side is very good. On the long wave side, there is, however, a pronounced step, indicating an additional maximum of emission somewhere around 4000A.

This discrepancy between calculated and experimental values on the long wave side demands further investigation. We suspect that it is due to some reaction with residual gas in the cell, or possibly to some anisotropy in the alkali metal film. In support of this belief, we give in Fig. 2, several experimental curves for potassium on

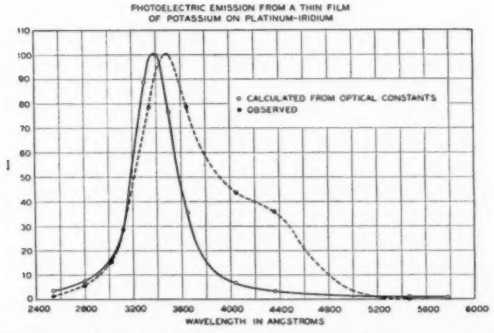

FIG. 1. Calculated and observed photoelectric emission from thin film of potassium on platinum-iridium.

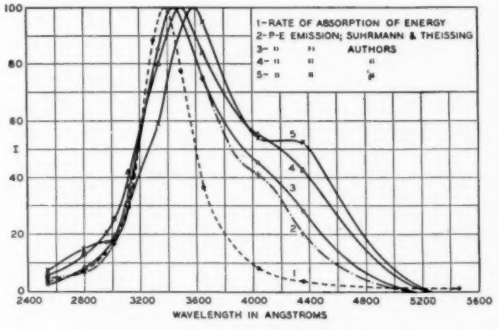

FIG. 2. Calculated and observed photoelectric emission from thin film of potassium on platinum.

[4] Ives and Briggs, "The Photoelectric Effect from Thin Films of Alkali Metal on Silver," Phys. Rev. **38**, 1477 (1931).

platinum (together with the computed curve, as tabulated in Table IV) where the same phenomenon is exhibited. By giving data on platinum we are able to give not only our own results but to quote a curve by Suhrmann and Theissing,[5] in which the same step appears (although in drawing their curve they disregarded the indication of a step form and made a smooth plot). It will be seen that these observations on different cells show the additional maximum to varying degree, and also indicate that the shift of the principal maximum toward longer wave-length, as already noted, goes hand in hand with the magnitude of this enhanced emission. We also quote, in this connection, the recent work of Cashman and Huxford on the *Photoelectric Properties of Pure and Gas Contaminated Magnesium*[6] in which they observe the development of an additional maximum on the long wave side together with a shift toward longer wave-lengths of the principal maximum, on admitting hydrogen gas, producing distortions of the pure magnesium curve of precisely the character we observe in the present instance. We feel, therefore, that further experimental work aimed at the complete exclusion of all gas contamination may well be expected to bring the observed results into very close agreement with the computed.

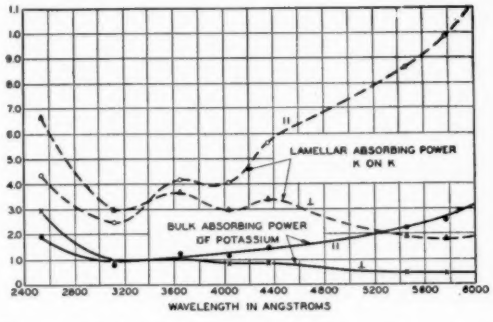

FIG. 3. Calculated bulk and surface layer photoelectric emission from potassium.

THE BEHAVIOR OF BULK POTASSIUM

The optical theory here tested with gratifying results for the case of thin films should apply in its complete form to the bulk alkali metal.[3] When we use our optical constants to compute for this condition, we obtain for the expected emission the values shown in Fig. 3 for the two cases of, (A_L) emission being restricted to the surface (lamellar absorption), and (A_B) with no such restriction (bulk). According to our line of thought, the first case should most nearly represent the facts, but it will be seen that while there are indications of an incipient maximum near 3400A for this case, the outstanding fact is that the maximum has practically disappeared for both A_L and A_B, and that the characteristic large ratio of emission for the two planes of polarization no longer exists in either form of absorption.

Experimentally, spectral maxima are almost always found with the bulk alkali metal, rather uniformly at longer wave-lengths than that here computed and found experimentally for thin films. Frequently these maxima are located at the wave-length characteristic of the hydride (4400A). Against these results are, however, to be placed several experiments, notably those of Fleischer and Dember[7] which indicate that a very fresh, uncontaminated potassium surface exhibits no maximum, and that maxima develop later, due probably to gas contamination. After the lapse of time we would, therefore, have thin films of some alkali metal compound overlying a base of alkali metal, which is similar to the condition purposely produced in our experiments with alkali metal film and platinum base. We feel that the optical theory is so fully confirmed by the subject matter of this paper, that the occurrence of spectral maxima of emission with the bulk alkali metals must be regarded actually as thin film phenomena. In the case of the sodium-potassium alloys in which Pohl and Pringsheim first observed the strong vectorial effect, our interpretation would be that we have here a surface layer of atoms or molecules of different composition (or possibly orientation) from the mass of liquid below.

[5] Suhrmann and Thiessing, Zeits. f. Physik **55**, 701 (1929).
[6] Cashman and Huxford, Phys. Rev. **48**, 734 (1935).
[7] Fleischer and Dember, Zeits. f. tech. Physik **7**, 133 (1926).

The Measurement of the Aberrations of a Microscope Objective

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(Received March 5, 1936)

After outlining the methods that have been employed for measuring the spherical aberration of microscope objectives, a new method is described in which a thin metal plate having a small hole in it is mounted between the objective and the microscope tube in such a way that the hole may be traversed across the middle of the objective aperture by a micrometer screw. The point at which the small beam of light thus isolated intersects the image plane is located by means of a filar micrometer. This gives a direct measure of the transverse aberration of that zone of the lens which contains the sliding hole, from which either the optical path difference or the ordinary longitudinal aberration of the zone can be readily computed. The aberration is then plotted graphically against the numerical aperture of the zone. The limitations and the precision of the method are discussed. The Foucault knife-edge test was found to be readily adaptable to the measurement of the chromatic aberration of a microscope objective. Typical curves for all these tests are given.

IN the current practice of today, microscope objectives are usually tested by observing visually the image of a suitably chosen test object. At one time various diatoms were the accepted standards, and even now, resolution of the rows of dots in a diatom is commonly regarded as the criterion of excellence of a microscope objective. In more recent years, the "star" test, performed with the help of minute dust holes in a chemically deposited silver film, has come into general use, and the ruled Abbe test-plate is also available. Nevertheless the decision remains one of visual estimation, although the star test rightly applied will enable an observer to detect traces of spherical aberration that are actually too small to be significant.

None of these visual tests can be made to yield a definite figure of merit by which one objective may be compared with another without relying on the judgment of an individual. Such a figure is best determined by measuring the actual spherical aberration of the objective, and calculating from it the curve of optical path difference against aperture relative to the determined "best focus." This may then be compared at once with the Rayleigh quarter-wave limit and a significant figure of merit thereby obtained.

Moreover, a curve of spherical aberration, if accurately determined, is of the greatest help to the lens designer and manufacturer, for without it he has no means of telling whether his product is as good as his design. However, once a design is accepted and put into production, the familiar star test is then quite suitable for passing or rejecting objectives in the factory.

PREVIOUSLY PROPOSED METHODS

(a) The Hartmann test

This has been adapted successfully to a microscope objective by Martin,[1] who has described in detail the testing of an 8 mm objective of N.A. = 0.65. The diaphragm used by him was 10 mm diameter; it would be scarcely possible to construct a smaller diaphragm with sufficient accuracy, and in any case the holes in the diaphragm would need to have a diameter of about 0.9 mm diameter to comply with Hartmann's rule that the hole diameter should be 1/200 of the image distance.

(b) The Beck test

Beck[2] has described a method for the precise location of zonal foci by inserting behind the lens a diaphragm containing two oppositely inclined slots lying in a single zone of the lens. The focus of this zone is then located by means of a second microscope arranged to slide along the axis of the test objective, the exact focus being at the point where the cross-shaped image appears symmetrically divided. On account of the length of these slots, the separate zones considered in this test are necessarily rather broad.

(c) The Hartridge test

To avoid the necessity of constructing a diaphragm small enough to insert behind the test objective, Hartridge[3] has suggested that a

[1] Martin, Trans. Opt. Soc. **23**, 75 (1921).
[2] Beck, Trans. Opt. Soc. **28**, 37 (1926).
[3] Hartridge, Proc. Camb. Phil. Soc. **21**, 29 (1922).

narrow beam of light could be isolated by a small sliding aperture placed below a well-corrected substage condenser. The "object" is a small silver filing, and the sideways motion of the image is measured as the small hole is moved step by step across the condenser aperture.

(d) The Twyman interferometer method

A modification of the Hilger Lens-Testing Interferometer for testing microscope objectives has been described by Twyman.[4] Very little has been published as to the effectiveness or usefulness of this instrument, but there is no doubt that it is as valuable a tool as the other Hilger interferometers.[5] The chief objection to this method is undoubtedly the inevitable high cost of the apparatus.

A Suggested New Test for Spherical Aberration

After trying a number of possible procedures for measuring spherical aberration, it soon became clear that on account of the small size of the lenses to be tested, methods in which a diaphragm is employed having holes in several zones of the objective (such as the Hartmann or Gardner-Bennett test) could never be really satisfactory because of interference effects between light from adjacent holes. An attempt was therefore made to traverse a single hole across the objective aperture to isolate a series of narrow pencils of light in succession, instead of simultaneously as in the other tests. An adapter was constructed to fit between the test objective and the tube of the microscope, in which a thin brass plate having a 0.5 mm hole in it could be moved across the objective aperture by means of a micrometer screw, a spring being used to keep one end of the sliding plate pressed against the pointed tip of the screw. The object was the usual illuminated "star" in a silver film deposited chemically on a cover glass, a whole series of these silvered cover glasses being prepared so that the best cover thickness could be used in every case. The transverse movements of the beam of light thus isolated were measured by an ordinary filar micrometer eyepiece in some chosen focal plane in the neighborhood of the image.

It speedily became apparent that very little accuracy could be expected in this simple way. The objective used for the first trials was a 4 mm, 0.85 N.A. of old pattern, a previous inspection having shown that a cover thickness of 0.20 mm gave the best star image with the tube length employed. As light source, a ribbon filament lamp and gelatine filter were used; a monochromator was tried but it was found to give too little light. There was no need to isolate a single star because an observer would never be likely to confuse his star with the others in the field while making settings. Care was necessary in choosing the focal plane for this test, it being found best to adopt that plane in which the spot of light fell on the optical axis both for the center zone and for a zone almost at the extreme edge of the lens aperture. In this way the graphs could be kept compact, and the possibilities of improving the image by a slight focal shift could be best utilized.

The real difficulties however were twofold; firstly, those due to vibration or elastic deformations of the apparatus during a series of settings, and secondly, the difficulty of making settings with a single cross hair on a large diffuse Airy disk, the diameter of the disk being 0.38 mm, which was about twice the greatest displacement that had to be measured. The first of these difficulties was overcome by the most rigid clamping of all parts of the apparatus, especially the filar micrometer eyepiece. It was found that the most sensitive part of the apparatus and the greatest source of erratic movements, namely, the sliding aperture plate, could not be clamped because any method of clamping invariably introduced some stresses which would gradually produce a slight unwanted movement of the spot of light. With an objective magnifying 40 times and a sensitivity of observation of about 5 microns, any lateral motion of the objective of as little as 0.1 micron would be detectable. As it was necessary to move the sliding aperture from one zone to another between settings, a separate knurled head shown in Fig. 1 was mounted independently on the table, which was connected to the micrometer screw of the sliding aperture by a short rod and two very free universal joints.

[4] Twyman, J. O. S. A. **7**, 635 (1923); also Trans. Opt. Soc. **24**, 189 (1923).
[5] Smiles, J. Roy. Mic. Soc. **50**, 307 (1930).

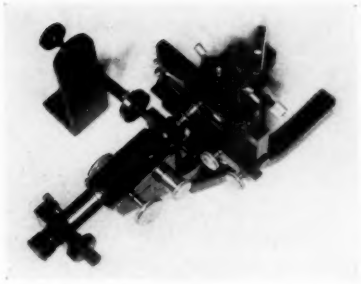

FIG. 1. Microscope equipped with sliding aperture behind the test objective, and a filar micrometer eyepiece.

In this way unwanted movements of the apparatus could be reduced below the precision of settings. It would be an advantage to have a similar extension control to the filar micrometer eyepiece. Probably an ideal apparatus for these measurements should be specially constructed, each unit being separately mounted on a very rigid lathe-bed type of bench. When oil-immersion objectives are being tested, the test slide must be clamped to the microscope stage to prevent its being dragged out of focus by the surface tension of the oil.

To improve the accuracy of settings, it was necessary to provide a fiducial mark which would move with the Airy disk. The device suggested by Chalmers,[6] and later independently by Cotton,[7] of using two very small holes close together was tried with considerable success. For this, the single hole of 0.5 mm diameter was replaced by two holes each 0.15 mm diameter, separated by a distance of 0.5 mm, both holes lying on the same horizontal diameter of the objective. Under these conditions the Airy disk became covered with a series of vertical interference fringes, any one of which could be used as a fiducial mark when measuring the transverse displacements of the spot of light. In this way it was found possible to make and repeat measurements of the transverse aberration T (Fig. 2) to an accuracy of about 0.003 mm. The separation of the holes had to be so small that there was effectively no variation in the aberration between them, and for more precise work it was found possible to use a pair of holes each 0.1 mm diameter at a separation of 0.25 mm.

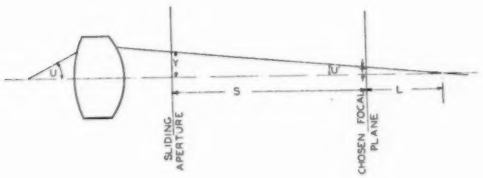

FIG. 2. The relation between longitudinal and transverse aberration.

THE CURVE OF APERTURE AGAINST ABERRATION

In performing a test with this apparatus, a number of positions of the sliding aperture plate were chosen, more or less equally spaced, and at each position a setting was made on the central band of the interference system with the filar micrometer. The two limits of the aperture were readily observed, for at each limit one of the small holes became cut off before the other, causing the interference fringes to vanish, thus leaving the plain Airy disk due to the other hole. The observed points were plotted on graph paper, with the aperture positions as ordinates and the filar micrometer readings as abscissae. The optical axis was marked on the graph, midway between the two edges of the aperture, and on this axis was marked the centroid of the curve as nearly as could be determined. If the lens was perfectly centered and if the measurements were all quite accurate, this point would be where the smooth curve drawn through the observed points crosses the optical axis line, and the upper and lower halves of the curve would have point-symmetry about this center. The observed points above the axis were then pricked through on to a separate piece of graph paper, letting the centroid fall on the origin, after which the first curve was rotated in its own plane through 180° and the points below the axis were pricked through on top of the other points. A fairly smooth curve could then be drawn through all the observed points.

This gave the curve of transverse aberration for the arbitrarily chosen focal plane. To convert this to the more familiar curve of longitudinal aberration L, the transverse aberration T at each

[6] Chalmers, Proc. Opt. Conv. (1912), p. 156.
[7] Cotton, Physica **1**, 274 (1921); also Rev. d'optique **3**, 401 (1924).

point was multiplied by $s/(y-T)$ (Fig. 2). The ordinates of this curve may be the y observed directly on the sliding plate micrometer, or they may be replaced by the corresponding values of the numerical aperture, since if m is the magnification, the well-known sine theorem gives (Fig. 1):

$$\text{N.A.} = \sin u = m \sin u' = my/s \text{ (approx.).} \quad (1)$$

The magnification m must be measured with the sliding plate removed, the tube length being kept rigidly constant. We may regard m as being the same for all zones, provided we may assume that the objective is reasonably well corrected for the sine condition.

Calculation of Optical Path Differences

To determine the real seriousness of the aberration found in any objective, it is necessary to compare the maximum residual path-difference at the best focus with the Rayleigh quarter-wave limit. This residual path difference may be determined by calculating the path-difference curve for some particular focal plane such as the one used in the experimental determination of the transverse aberration, by means of the well-known approximate relation:

$$P = \frac{1}{s}\int_0^y T\,dy. \quad (2)$$

For our purposes it is sufficiently accurate to replace the integral sign in this expression by the direct summation

$$P = \frac{\delta y}{s}\sum_0^y (T) \quad (3)$$

provided δy is sufficiently small. Even if δy is as large as 0.2 mm, the direct summation will give a curve of path-difference which is quite as accurate as the original data will permit.

If these values of P are now plotted against y^2, the position of the best focus may be at once determined by finding the closest pair of parallel straight lines which just enclose the curve, and by drawing a straight line through the origin parallel to this pair of straight lines. The slope of this line is a measure of the shift δs of the focal plane, since the path-difference in wave-lengths caused by a change in focus δs is given by

$$P = y^2 \cdot \delta s/2s^2\lambda. \quad (4)$$

Also, the horizontal distance between the two parallel bounding lines is a measure of the maximum residual path-difference at the best focus, which may be read off the graph and at once compared with the Rayleigh quarter-wave limit.

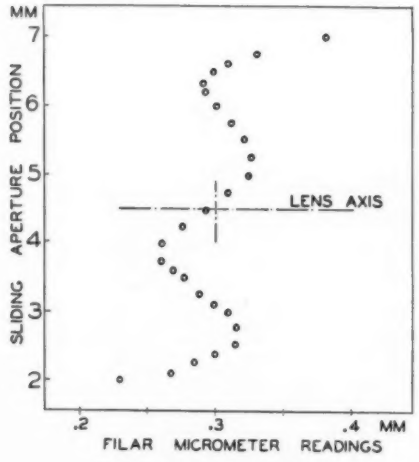

Fig. 3. The observed points.

An Example of This Method

The measured transverse aberration of the 4-mm objective of N.A. = 0.85 used in these first trials, at an arbitrarily chosen focal plane known to be near the best focus, was as shown in Fig. 3.

Table I. ($s = 145$ mm.)

y (mm)	T (mm)	$L = Ts/(y-T)$ (mm)	ΣT (mm)	P (λ)	y^2
0	0	(?)	0	0	0
0.2	0.014	10.9	0.014	0.032	0.04
.4	.026	10.1	.040	.092	.16
.6	.034	8.7	.074	.170	.36
.8	.032	6.0	.106	.244	.64
1.0	.024	3.6	.130	.300	1.00
1.2	.014	1.7	.144	.332	1.44
1.4	.004	0.4	.148	.341	1.96
1.6	−.005	−0.5	.139	.320	2.56
1.8	−.012	−1.0	.127	.292	3.24
2.0	−.005	−0.4	.122	.281	4.00
2.2	.016	1.1	.138	.318	4.84
2.4	.046	2.8	.184	.425	5.76
2.6	.095	5.5	.279	.642	6.76

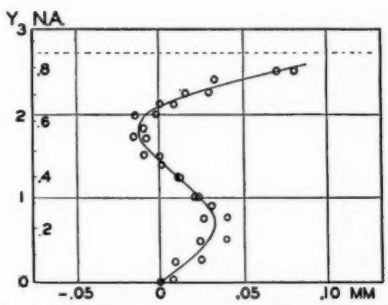

FIG. 4. The transverse aberration curve.

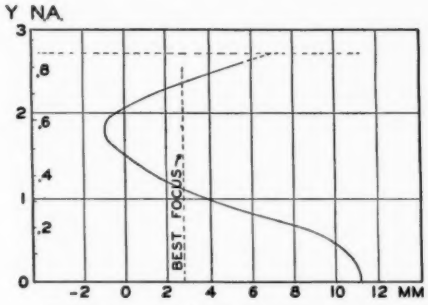

FIG. 5. The longitudinal aberration curve.

In Fig. 4 are shown the observed points superposed as described above, with the best smooth curve drawn through them. The abscissae, T, of this curve, measured from a vertical line through its zero point, were then read off and tabulated for each successive 0.2 mm in the value of y (Table I).

In the third column of this table are the corresponding values of the longitudinal aberration L, shown graphically in Fig. 5. In column 4 are the cumulative sums of the values of T, which become the path-differences in column 5 on multiplying them by $\delta y/s\lambda$. In this case, $\delta y = 0.2$ mm, $s = 145$ mm, and $\lambda = 0.0006$ mm. The curve of path-difference against y^2 is shown in Fig. 6, together with the closest-fitting pair of parallel lines. The horizontal distance between these lines is found to be 0.36λ, which is 1.4 times the Rayleigh limit of $\lambda/4$. The position of the best focus is found, by means of Eq. (4), to be at a distance $\delta s = 2.36$ mm from the focal plane chosen in the original experiment; it is indicated on the longitudinal aberration curve in Fig. 5.

The magnification was found to be 45.5; by means of this the numerical aperture scale has been added to the graphs, and it will be noted that the curves have been extended to the N.A. of 0.85 which is the limit of the objective aperture. It is impossible to locate the focus of the extreme marginal zone by this method because as soon as one of the two holes moves out beyond the rim of the lens aperture, all interference ceases and the interference fringes on which settings are made vanish. In fact, the largest zone that can be used in this test has a diameter which is less than the clear aperture of the objective by the spacing of the holes in the sliding plate.

ACCURACY OF THE METHOD

It is found that, provided the apparatus is clamped sufficiently rigidly to prevent unwanted movements of the various parts, it is possible to make and repeat settings with the filar micrometer to two or three microns. The greatest source of error is lack of rigidity of the apparatus, the presence of which can be at once detected while making settings by frequently returning the sliding plate to its axial position and observing whether the filar micrometer reading repeats itself accurately. This error of 3 microns in the readings corresponds to an error of $0.003s/y$ in the longitudinal aberration, which amounts to 0.17 mm for $s = 145$ mm and $y = 2.6$ mm at the marginal zone of the 4-mm objective used in the tests described above. The error in longitudinal aberration increases as the zone-radius y becomes smaller, finally reaching about 2.2 mm at $y = 0.2$ mm. Greater ease and precision in the filar micrometer settings could be secured by the use of a pair of parallel cross wires separated by about the width of the bright interference bands on which settings are being made, instead of the single cross wire usually supplied in a filar micrometer eyepiece.

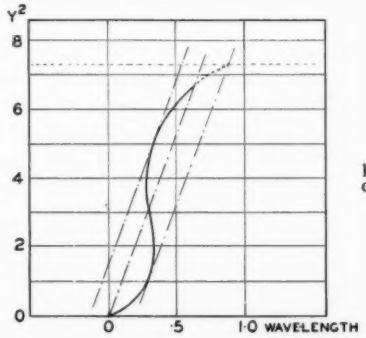

FIG. 6. The path difference curve.

Test for Chromatic Aberration

For this test, an isolated star in a silver film was illuminated by focusing upon it an image of the exit pinhole of a small monochromator, by means of the substage condenser. Light from an arc lamp was used, as the light from a projection lamp was found to be scarcely bright enough for reliable observations. It was necessary to refocus the substage condenser at each wave-length used because of the chromatic aberration of the condenser. A simple sliding knife-edge holder was constructed and placed behind the microscope tube, so that the knife blade could be moved along the axis of the system, its position being read off a simple scale to an accuracy of 0.1 mm. A setting was made at each wave-length by choosing that longitudinal position of the knife-edge at which the shadow was seen to jump suddenly across the center of the lens aperture for a very small forward movement of the knife-edge, which phenomenon was assumed to mark the paraxial focus of the objective. With a little practice, these settings could be repeated within a few tenths of a millimeter. The curve obtained for the 4-mm 0.85 objective is shown in Fig. 7.

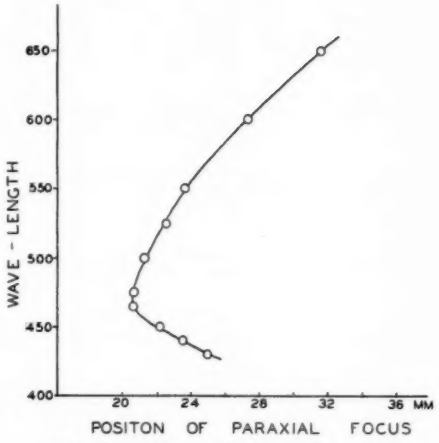

FIG. 7. The chromatic aberration curve.

JUNE, 1936 J. O. S. A. VOLUME 26

Transmission Curves for the New Polarizers

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(Received May 18, 1936)

SEVERAL materials are now available, commercially, as substitutes for Nicol prisms. These open up new possibilities for the application of polarized light. The accompanying curves of transmission of two of these may be of interest in connection with experiments where the efficiencies of such polarizing equipment are important.

Two types of material are on the market, one in the form of film with the active material dispersed throughout its thickness, the other as a plate coated with a layer of the active material. The plates and films represented here are typical of the material now available.

The curves, Fig. 1, are for the transmission of two plates (or films) in series in a beam of unpolarized light. In one case they were arranged with the polarizing azimuth crossed, in another they were arranged parallel. I believe the curves are self-explanatory. For the films crossed, the transmission around 4500A reaches $\frac{1}{2}$ percent, from 5000 to 6700A it is nearly zero.

The crossed films could be used as a filter to isolate the visible radiations from the infrared. This filter could even be adjusted to vary the ratio of the visible to infrared light transmitted.

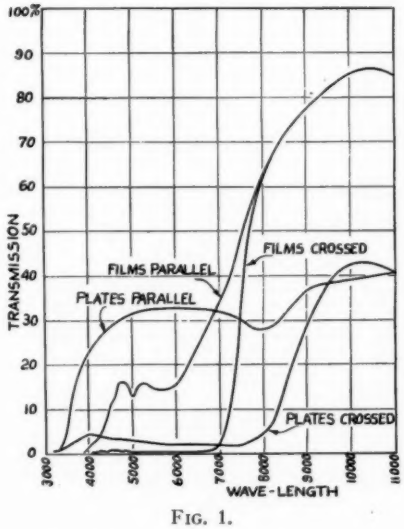


FIG. 1.

Scattering of Daylight in the Sea

C. L. Utterback and Wilhelm Jorgensen, *Physical and Oceanographic Laboratories, University of Washington, Seattle*

(Received March 9, 1936)

Measurements of the scattering and transmission of three spectral bands of visible solar radiation have been made in the San Juan Archipelago. Ratios of the scattered light to the transmitted light of each spectral band have been computed and the results shown graphically.

A CONSIDERABLE amount of work has been done during the last few years on the transparency of natural water bodies,[1] but very few observations have been made to determine their scattering properties. Pettersson[2] has made a few observations on scattered light in Gullmar Fjord using a submerged incandescent lamp as a source with the scattered light received by a photoelectric cell placed near the source, but out of the path of the transmitted beam.

The present study was undertaken to investigate the transmission of three spectral bands of visible solar radiation through the water and to investigate the light scattered vertically upward in these same bands. The experiments were conducted in the waters of the San Juan Archipelago in the late summer of 1935. A station was selected such that the depth of the water was about 40 meters and the observations reported were made on a long high tide.

Apparatus

In order to measure the transparency of the water to bands of solar radiation transmitted through each of the three color filters the submarine photometer shown in Fig. 1 was constructed.

A cylindrical steel tube, provided with a flange at each end, was closed at the upper end with a piece of boiler plate. This plate was constructed, as shown, to admit lead wires through a water tight joint. These wires connected the cell with a d'Arsonval galvanometer, which had a sensitivity of $98 \cdot 10^{-6}$ microampere. The other end of the cylindrical tube was closed by means of a glass window. The window and the boiler plate were mounted on gaskets making water tight joints. The cell was mounted on an adjustable support in such a way that the color filters could be placed between the cell and the window. Some calcium chloride was placed in the tube to absorb any water vapor present.

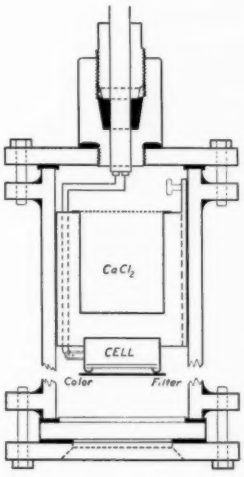


Fig. 1. Submarine photometer.

Three color filters were used. One had a fairly narrow transmission band in the blue, another a band in the green and the third a band in the red portion of the spectrum. The effective wavelengths[3] assigned to these transmission bands are 4800A, 5300A, and 6000A.

A deck photometer, provided with a duplicate set of filters, was used to measure any variations in intensity of the light incident on the surface of the water.

[1] For a bibliography of papers published on transparency see—W. R. G. Atkins, J. du Conseil (VII) **2**, 171 (1932); W. R. G. Atkins and H. H. Poole, Phil. Trans. Roy. Soc. Dublin **B478**, 129 (1933); Geo. L. Clarke and R. H. Oster, Biol. Bulletin **67**, 59 (1934); Hans Pettersson, Göteborgs Kungl. Vetenskaps-Och Vitterhets Samhälles Handl. **B4**, 5 (1934); E. Allen Williams and C. L. Utterback, J. Opt. Soc. Am. **25**, 384 (1935).

[2] Hans Pettersson, J. du Conseil (X) **1**, 47 (1935).

[3] C. L. Utterback and Wilhelm Jorgensen, J. du Conseil (IX) **2**, 197 (1934).

Procedure

With one of the filters between the cell and the window and with the window downward the photometer, suspended by a cable, was lowered until the window was 6 inches beneath the surface of the water. The current was measured with the photometer in this position. It was then lowered, by five-meter intervals, as deep as the intensity of the upward scattered light could be accurately recorded. The photometer was next raised by the same five-meter intervals and the current measurements repeated. The photometer was then suspended on the cable so that it again was hanging vertically but this time with the window upward instead of downward as before. A similar series of measurements gave the intensity of the downward transmitted light at the different depths. These measurements were repeated for each of the filters and, at all times, simultaneous observations of the intensity of the light incident upon the surface were made with the deck photometer, screened with the proper filter.

Results

The relative intensities of the downward transmitted light and of the upward scattered light for each filter were plotted on a logarithmic scale against the depth. From these graphs the data in Table I were computed.

In each series of measurements the intensity of the light 6 inches below the surface was taken as 100 percent. The intensity of the transmitted light and the intensity of the upward scattered light, at the different depths, was computed in terms of the 6-inch subsurface light. The values of the relative intensities, for each band, are given in column 2 under the proper wave-length. The logarithm of these relative intensities are in column 3 and the intensities, expressed as percent of that 6 inches below the surface, are given in column 4.

The values of the ratio of the scattered light to the transmitted light have been computed for each color and are shown in Fig. 2.

Discussion

The observations were made under very ideal conditions. The surface of the water was exceedingly smooth. There were no clouds in the sky and the intensity of the light incident upon the surface of the water was very constant during the observations. The time required for each series of measurements was approximately ten minutes.

The comparison of the scattered light to the transmitted light shows variations which, no doubt, are due to the nature of suspended matter and organisms in the water. Particles and organisms are known to accumulate at different depths. There is evidence of diurnal migration of plankton. In these experiments the transmission and scattering data were not taken simultaneously, so it is possible that the content of the

Table I. *Showing the relative intensities of the downward and upward traveling light.*

Depth (meters)	Blue. λ = 4800A			Green. λ = 5300A			Red. λ = 6000A		
	Relative Intensity I	$\text{Log}_e I$	% of I at 0.15 m	Relative Intensity I	$\text{Log}_e I$	% of I at 0.15 m	Relative Intensity I	$\text{Log}_e I$	% of I at 0.15 m
				Downward Traveling Light					
0.15	18500	9.83	100	72100	11.19	100	156000	11.96	100
5	6990	8.85	38	29300	10.29	41	15000	9.62	9.6
10	2620	7.87	14	11800	9.38	16	1980	7.59	1.3
15	992	6.90	5.4	4780	8.47	6.6	340	5.83	0.22
20	372	5.92	2.0	1920	7.56	2.7	73.2	4.29	0.047
25	141	4.95	0.76	781	6.66	1.1	19.9	2.99	0.013
30	—	—	—	321	5.77	0.44	6.17	1.82	0.0039
				Upward Traveling Light					
0.15	351	5.86	100	713	6.57	100	347	5.85	100
5	145	4.98	41	347	5.85	49	90.3	4.5	26
10	58.7	4.07	18	161	5.08	26	23.6	3.16	6.8
15	22.9	3.13	6.5	71.7	4.27	10	6.18	1.82	1.8
20	8.59	2.15	2.4	30	3.40	4.2	1.63	0.49	0.47
25	2.89	1.06	0.82	12.1	2.49	1.7	—	—	—
30	0.91	−0.09	0.26	4.81	1.57	0.67	—	—	—

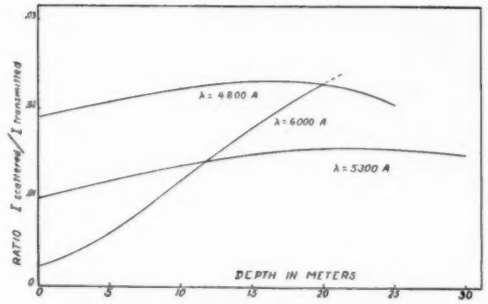


FIG. 2. Ratio of scattered light to transmitted light.

different layers altered some between series of observations.

The relatively low scattering of the red light near the surface may be explained to some extent by the absence of organisms which are found at greater depths and which are known to reflect the longer wave-lengths. The low value of the ratio of scattered light to transmitted light near the surface may be explained, in part, by the fact that the longer wave-lengths of the light transmitted through the red filter are absorbed largely in the upper layers of water.

The results obtained in these measurements are not entirely in agreement with the results obtained by Pettersson.[2] The values of scattering coefficients of light obtained from subsurface measurements may depend greatly on the manner of making the measurements. The amount of energy scattered in a layer of water depends not only on the amount of energy which has been transmitted to that layer but also on the spectral distribution of the energy and the scattering particles in the water. Therefore, scattering coefficients obtained from an artificial source lowered to various depths will differ from those obtained by the use of solar radiation.

An artificial source is uniform in intensity and spectral distribution at any depth at which it may be placed, whereas in the case of sunlight the spectral distribution undergoes very great changes as the depth increases. The results of Hulburt[4] and of Atkins and Poole,[1] shown in Table II and obtained by interpolation of their data, illustrates the variation in the spectral distribution of light in different waters. Hulburt's results were obtained by spectrophotometer measurements on a 4 meter column of pure sea water from which all suspended matter had been removed, while those of Atkins and Poole were obtained at the International Hydrographic Station E-1.

TABLE II. *Percent of transmitted light computed from the data of other observers.*

DEPTH (meters)	PERCENT OF TRANSMITTED LIGHT FROM HULBURT'S DATA 4800A	5300A	6000A	FROM ATKINS AND POOLE'S DATA 4800A	5300A	6000A
0	100	100	100	100	100	100
10	85.8	78.7	19.2	23.3	20.8	10.7
20	73.5	61.9	3.7	5.4	4.3	1.1

The values of the percent of transmitted light in Table II, when compared with the values given in Table I, show that in these waters the long wave-lengths of the spectrum are largely, but not equally, absorbed in the upper layers.

It is also apparent that the blue region of the spectrum is the least absorbed in the off shore waters where the measurements were made by Atkins and Poole. However, in the case of the inshore waters in the San Juan Archipelago, the absorption is least in the green portion of the spectrum. This is in agreement with the results of Knudsen[5] who found the maximum transmission in the green in the waters of Nyborg Fjord. These results illustrate the fact that light from an artificial source, lowered to different depths, would not be composed of the same wave-lengths as solar light transmitted to the same depths and hence the spectral distributions of the scattered light would not be the same. The variation between scattering as obtained with an artificial source and as obtained with transmitted solar light may, to some extent, be due to a different distribution of swimming organisms under the influence of a fairly intense submerged light.

A rapid examination of water bodies, including simultaneous observations of scattering and transmission of narrow spectral bands, may afford valuable information on the influence of light on growing organisms which are below a turbid surface, or any subsurface layer. Such an examination will be undertaken in the summer of 1936.

[4] E. O. Hulburt, J. Opt. Soc. Am. **23**, 170 (1932).

[5] M. Knudsen, Conseil Internat. Expl. Mer. Pub. de Circons, 76.

A Portable Blocking-Layer Photo-Cell Pyrometer

GROVER F. HUBING, *Department of Physics, University of Cincinnati*

(Received January 16, 1936)

THE subject of this paper utilizes the ratio of the current outputs of a photo-cell when exposed to total radiations and then to radiations transmitted by a light filter. The radiation sensitive element is a direct action, blocking-layer photo-cell requiring no external e.m.f.

Previous photoelectric pyrometers,* while giving excellent results, contained several cells, filters and electroscopes and were not suited for use in technical practice. The pyrometer described incorporates the advantages of photoelectric methods into a small and easily operated instrument.

If $f_S(\lambda)$ is the monochromatic spectral response of the photo-cell and $f_F(\lambda)$ is the monochromatic transmission of the light filter, the current output of the cell in amperes is

$$I_T = 0.045 C_1 \int f_S(\lambda) \cdot \lambda^{-5} \cdot e^{-C_2/\lambda T} \cdot d\lambda \qquad (1)$$

when exposed to the total radiations and

$$I_f = 0.045 C_1 \int f_S(\lambda) \cdot f_F(\lambda) \cdot \lambda^{-5} \cdot e^{-C_2/\lambda T} \cdot d\lambda \qquad (2)$$

when exposed to radiations passing through the filter. The current ratio R is then

$$R = \frac{0.045 C_1 \int f_S(\lambda) \cdot \lambda^{-5} \cdot e^{-C_2/\lambda T} \cdot d\lambda}{0.045 C_1 \int f_S(\lambda) \cdot f_F(\lambda) \cdot \lambda^{-5} \cdot e^{-C_2/\lambda T} \cdot d\lambda}, \qquad (3)$$

where $C_1 = 3.70 \times 10^4$ watts per cm^2,
$C_2 = 14{,}325$ for λ in μ,
T = absolute temperature,
0.045 = constant of the photo-cell.

Because of the difficulty of expressing the spectral response and the filter transmission mathematically, a graphical solution was applied to Eq. (3).

The ordinates of the distribution curve of a perfect radiator at an assumed temperature were multiplied by those of the cell response curve at corresponding wave-lengths, the values plotted and the curve integrated. The current output of the photo-cell when exposed to total radiation is proportional to the area under this curve, which is curve A in Fig. 1.

The current output of the cell when exposed to filtered radiations is proportional to the area under curve B which was obtained by multiplying the ordinates of curve A by those of the transmission curve of the filter at the same wave-lengths and integrating.

By solving for the ratio at different temperatures, the relation existing between the temperature of a black body and the current ratio was expressed as a curve as in Fig. 2.

A Corning light filter, having a single peak of transmission of 86 percent at 0.4750 micron, and absorbing radiations beyond 0.6500 micron was selected. The transmission curve of the filter was determined by spectrophotometric measurements.

When measuring the current ratio of tungsten at high temperatures, a change in the distance between the source and the photo-cell produced a change in the ratio, the temperature being held constant. This was concluded to be the result of deviations from strict linearity at the low illuminations encountered, either with the cell exposed to total radiations or to those passing through the filter, or both.

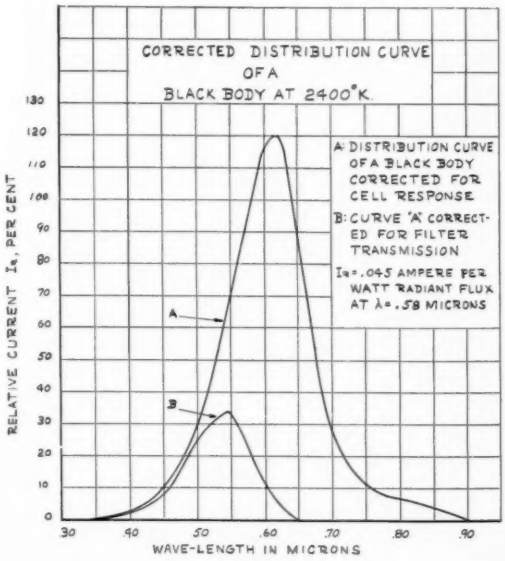


FIG. 1.

* Lindemann and Keeley, Proc. Phys. Soc. London **38**, 69 (1925); J. Sci. Inst. **2**, 177 (1925).

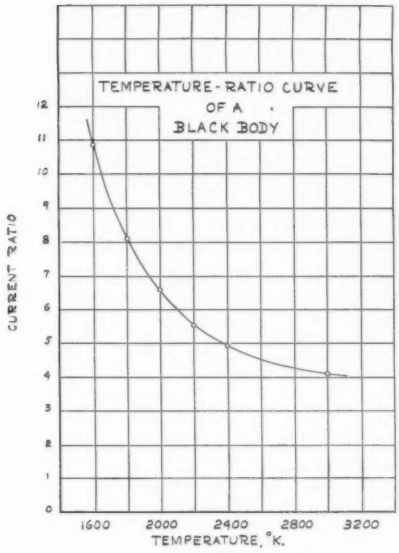


FIG. 2.

A grill of thin metal strips, mounted so that it could be revolved on its central axis, was placed before the photo-cell, providing a means of controlling the intensity of illumination. By revolving the grill, the intensity of total radiation upon the photo-cell could be adjusted to any assumed standard, thereby insuring all the readings of total radiations to be identical, regardless of the distance between the source and the photo-cell. If the above condition is satisfied, any deviation from strict linearity would be constant, and would not affect the usefulness of the instrument.

Fig. 3 shows the general arrangement of the instrument, with photo-cell *C*, grill *G*, and the filter *F* mounted so that it can be placed in front of the cell, allowing either total or selected radiations to be measured. The whole is mounted

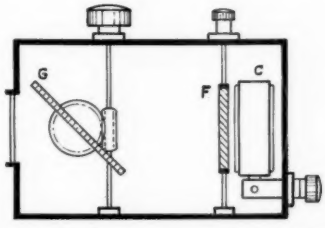


FIG. 3. Detail showing arrangement of photo-cell, filter and grill.

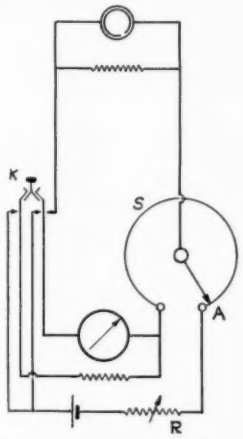


FIG. 4. Circuit diagram of null potentiometer for measuring current ratios.

in a small light and dust-proof box fitted with a glass window.

Fatigue in the photo-cell is either negligible or effects both readings alike, as it cannot be detected with the most sensitive measuring circuits, even during continuous operation of the instrument.

The ratios of the current outputs of the photo-cell were measured with the null potentiometer circuit shown in Fig. 4. The working current is adjusted to the operating standard by depressing the key *K* to insert the galvanometer into the working circuit, and adjusting *R*.

To operate the instrument with the sliding contact in position *A*, the grill is adjusted to produce a zero galvanometer reading, the filter placed before the photo-cell and the slide wire *S* adjusted to give again a zero reading. The ratio is read directly from the scale of the instrument which may be calibrated in terms of temperature.

Observations on an experimental black body, the temperature of which was measured with an optical pyrometer, indicated an error in the calculated temperature-ratio curve of 1 percent at 2000°K. By using a pointer and scale type galvanometer in the measuring circuit, temperature readings can be reproduced to within three degrees over the entire range of the instrument.

The author wishes to acknowledge the assistance of Dr. I. A. Balinkin in the development of the instrument.

A New Type of Interferometer

A. HOYT, *Gulf Research & Development Corporation, Pittsburgh, Pa.*

(Received March 25, 1936)

A description is given of an extremely simple interferometer which is easy to construct and adjust. It is admirably suited to the measurement of small displacements. It can be adjusted and used with either monochromatic or white light. Each of the small, light mirrors used needs have only one optical surface. The sensitivity can be increased by using multiple reflections without unnecessary loss of intensity, since the surfaces are all fully aluminized. Tests on the interferometer have shown that it is capable of detecting displacements of $\lambda/2n$ where λ is the average wave-length of the light used and n is the number of reflections from the moving mirror (nearly normal incidence). When $n=1, 2, \cdots, 7$, one fringe corresponds to $\lambda/2$, $\lambda/4$, $\cdots$, $\lambda/14$, respectively. Under best (easily attainable) conditions 14 reflections can be obtained, whence a shift of 1 fringe would correspond to $\lambda/28$. Extreme care in the preparation of optical parts might yield 20 reflections and a displacement sensitivity of $\lambda/40$ for each fringe shift. A compound multiple reflection interferometer is also described which gives twice the sensitivity mentioned above, i.e., roughly $\lambda/4n$ for each fringe shift. Selected plate glass can be used for a *single reflection* since only a *very small* mirror surface is required.

USE OF INTERFEROMETER IN AN EXTENSOMETER

THE indicating parts of some extensometers unfortunately require quite considerable operating forces. This is not necessarily true for interferometers so used and for a few other types of optical devices. A search for an extensometer which would operate without introducing resisting forces led to the development of the type of interferometer herein described. It can be used for many of the jobs usually handled by the interferometric method. Since the instrument is so simple, it can best be built to suit any particular problem at hand. For example, in the measurement of the thermal expansion coefficient of short quartz knife-edges some very small mirror adjusting units were made. It was found, as anticipated, that these same adjusting units were very useful in other applications of the Case I type of interferometer. This type of interferometer was also used as a "null indicator" on an aneroid barometer in the course of some subsequent experiments on the use of a barometer as an altimeter. It is probably unnecessary to mention other applications as many of them will be obvious to the reader. The instrument is well suited to the measurement of small displacements and is easily adaptable to cases in which the length-changes amount to any reasonable number of wave-lengths of light.

As an amusing example of an imaginary application, one might use the interferometer of Case I (with a white light source) to indicate very accurately exact whole revolutions of a wheel attached possibly to a micrometer screw.

The small displacements are detected by observing the shift of an interference fringe system. For slow motions of the fringe system, monochromatic fringes can be followed, but if any erratic shifts occur, white light fringes must be used so as to have a fiduciary reference line—the central achromatic fringe. The limitation of the number of white light fringes can be alleviated to some extent by the device of illuminating half the field with white and half with monochromatic light.

PRINCIPLES OF THE INTERFEROMETER

Case I. One reflection

As shown in Fig. 1 the narrow mirrors lie in the XY plane and close together. The reflecting surfaces used are two narrow strips adjacent to the crack between the mirrors. The mirror sur-

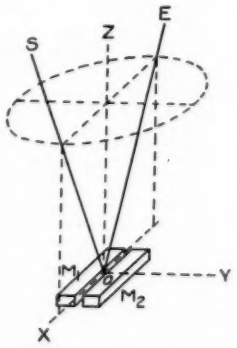


FIG. 1. Arrangement of optical parts of single reflection interferometer.

faces are coplanar if parallel light is used; otherwise they incline at a small angle with the XY plane. In this case the source S and eyepiece E must lie on opposite elements of a right circular cone about the Z axis, vertex at O. The plane SOE may have any azimuth, and SO need not equal OE.

The manner of production of fringes is shown in Fig. 2. If parallel light is used, the two parts

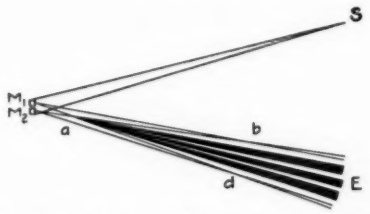


FIG. 2. Production of fringes by two mirrors.

of the wave front are made to superpose and interfere by the telescope. Otherwise the beams superpose on account of the inclination of the mirrors. The fringes appear in the region $abEd$. They are visible with the naked eye if the angle between M_1 and M_2 is sufficiently close to 180°. The use of a telescope magnifies the fringes and aids the eye in focusing on the proper region.

Consider a single reflection from each mirror M_1 and M_2. Let M_2 move parallel to itself in the $\pm Z$ direction. It can be shown that the path difference ΔP introduced by a motion Δz of M_2 is $\Delta P = 2\Delta z \cos i$, where i is the incidence angle. $\Delta P = \lambda$ corresponds to a shift of one fringe. Hence a shift of one fringe means that M_2 has moved a distance:

$$\Delta z = \lambda/(2 \cos i).$$

For this case only very minute mirrors are really required. Since reflecting faces about $1/3 \times 5$ mm are ample, these mirrors can be made very light. However, in practice it is easier to use somewhat larger mirrors than are absolutely required, screening off the unnecessary parts.

Small mirrors ($1/4'' \times 1/4'' \times 1/12''$), which were made optically good all the way to the edge, were obtained from J. W. Fecker, Pittsburgh, Pennsylvania. Only the top surface is polished, the other surface and edges are ground. Care was taken to avoid nicks in the edges to be placed adjacent in use.

One can regard this special case of the interferometer as an adaptation of the Fresnel mirrors. However that may be, the purpose and use of the two-mirror scheme here are quite distinct from the ordinary uses of the Fresnel mirrors.

Case II. Two opposed fringe systems—single reflection interferometer

By arranging a system of mirrors as shown in Fig. 3, it is possible to double the sensitivity by making two fringe systems appear in the eyepiece, so that they move in opposite directions. Mirrors m_1 and m_4 are mounted on the same movable beam B. One fringe system arises from m_1 and m_2 while the other system arises from m_3 and m_4. This arrangement has the further advantage that no fixed point of reference is required in the eyepiece. The edges used are adjacent to the line AA. No interferometer of this type has been built on account of lack of an immediate application in our business.

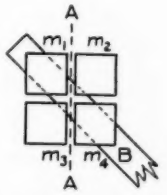


FIG. 3. Doubled sensitivity, with single reflection interferometer. m_1 and m_4 rest on the movable beam B, m_2 and m_3 are fixed. Edges adjacent to AA are used.

Case III. Multiple reflection interferometer

To obtain a multiplied sensitivity one must place the optical parts as shown in Fig. 4. The arrangement of parts is similar to that of Case I (Fig. 1), except that parallel light must be used and the source S and telescope E* must be confined to the XZ plane as must also the narrow space between the mirrors M_1 and M_2. (If S is a line source, the linear dimension must also be confined to this plane.) An additional mirror M_3 is added in the plane $z = C$ (constant), i.e., parallel to the XY plane and approaching

* The telescope objective should be put close to the mirrors for the best results.

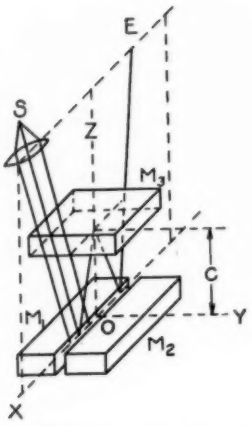


FIG. 4. Arrangement of mirrors for multiple reflection interferometer. The parts of the mirrors used are those adjacent to the XZ plane. Sensitivity to motion of M_2: one fringe shift means $\Delta z = \lambda/(2n \cos i)$.

the XY plane as closely as the chosen number of reflections and incidence angle require. As before, M_2 is movable approximately in the $\pm Z$ direction.

Part of the light producing the fringes reaches the eyepiece by the path $SM_1M_3M_1E$, the other part by the path $SM_2M_2M_2E$. These two light beams are reflected back and forth many times, each beam following along on its own individual mirror, entering the apparatus near one end and leaving at the opposite end. These multiple reflections take place near the adjacent edges of M_1 and M_2. The path difference ΔP introduced by a motion Δz of M_2 is given by:

$$\Delta P = 2n\Delta z \cos i,$$

where n = number of reflections from M_2, i = angle of incidence. Thus, a shift of one fringe measures the displacement Δz.

$$\Delta z = \lambda/(2n \cos i),$$

where λ = average wave-length of light used, and for normal incidence $\cos i = 1$.

In the actual set-up, stainless steel mirrors were used for M_1 and M_2 while M_3 was an aluminum-coated glass mirror. Since multiple reflections are used, it is advisable to use the best optical flats in order to get sharp fringes. However, reasonably good fringes can be obtained with quarter wave flats. In any case all the mirror surfaces

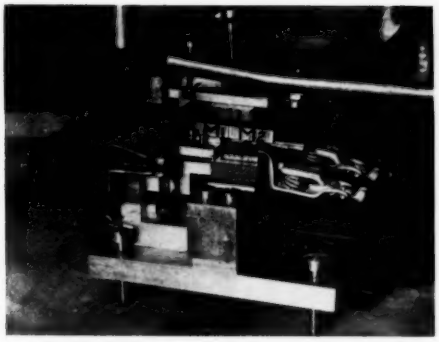


FIG. 5. Photograph of multiple reflection interferometer as first set up. With 10 reflections from the moving mirror, a shift of one fringe occurs for a motion of one-millionth of an inch.

can be opaque to light since no transmissions are involved in the instrument. This feature might be of some advantage with ultraviolet light in efforts to reduce absorption losses.

The photograph (Fig. 5) shows a very crude arrangement, but the fact that the adjustments can be made with such improvised apparatus shows that this particular arrangement is very practical. To obtain the fringes requires about 15 minutes for a single reflection, while the multiple reflection case requires only a couple of hours. In no case is it necessary to have recourse to monochromatic light in adjusting the apparatus, and its use did not prove to be much help in practice.

Case IV. Compound multiple reflection interferometer with two opposed fringe systems

An arrangement of mirrors shown in Fig. 6 can be used as an interferometer for which the sensitivity is double that of Case III. This doubling is effected by producing two separate fringe systems in the eyepiece which move in opposite directions when the mirror M_2 moves approximately parallel to itself in the $\pm Z$ direction. In this case both the top and bottom surfaces of M_1 and M_2 are fully silvered. An auxiliary mirror M_4 is added to produce multiple reflections in an additional pair of optical paths which are involved in producing the second fringe system in the eyepiece. The mirrors M_1 and M_2 must be parallel to M_3 and M_4 but need not be midway between them. M_1 and M_2 may

have their ends beveled off at the incidence angle so as to permit the two fringe systems to be contiguous. However, a slight tipping of M_3 or M_4 about an axis perpendicular to the crack between M_1 and M_2 will make the two fringe systems overlap to any desired extent. For a null reading instrument this pair of adjacent fringe systems removes the need of a fixed reference mark in the eyepiece.

Reflections are taken off both the top and bottom surfaces of M_1 and M_2. M_1, M_3 and M_4 are fixed. When M_2 moves, the path $SM_2M_3M_2E$ is lengthened while the path $SM_4M_2M_4E$ is at the same time shortened. A change in one of these paths produces a motion of one fringe system, and a change in the other produces a motion of the second fringe system in a direction opposite to that of the first. The number of reflections from the moving mirror in one case is n and in the second case is $n'=(n-1)$. When an apparent *relative* shift of one fringe appears, it is produced by a $\lambda/2$ change in each optical path; thus where

λ = average wave-length of light,
i = angle of incidence,
then $\lambda/2=\Delta P=2n\Delta z\cos i$,
$\lambda/2=\Delta P'=2n'\Delta z\cos i$.

Adding these equations, we see that a relative shift of one fringe measures a displacement Δz,

$$\Delta z=\frac{\lambda}{2(n+n')\cos i}=\frac{\lambda}{2(2n-1)\cos i}\approx\frac{\lambda}{4n\cos i}$$

(if n is large).

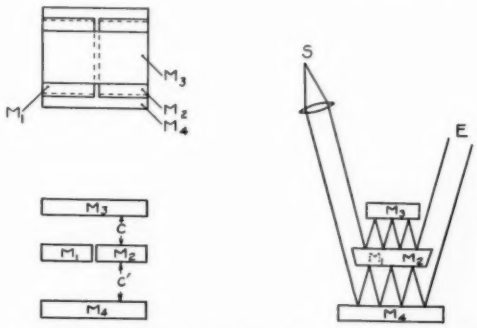


FIG. 6. Arrangement of mirrors for compound multiple reflection interferometer with two oppositely movable fringe systems. Sensitivity to motion of M_2: one fringe shift means $\Delta z=\lambda/[2(n+n')\cos i]$, where n and n' are the number of reflections from the top and bottom of M_2.

This further increased sensitivity is obtained by making the optical parts M_1 and M_2 fulfill further conditions, and by the use of still another auxiliary mirror (M_4). The former mirrors must (1) have parallel faces and (2) be of the same thickness. Probably this can best be fulfilled by cutting M_1 and M_2 from the same plane parallel plate. If only monochromatic light is to be used, condition (2) need not be very stringently fulfilled. With white light, however, the thicknesses of M_1 and M_2 ought not to differ by more than $\lambda/3$ or $\lambda/7$ approximately if $(n'+1)=n=10$ or 20, respectively. M_3 and M_4 must be good optical mirrors on one side only.

ADJUSTMENT OF THE INTERFEROMETER

Cases I and II. Single reflection interferometer

The adjustment is about as follows. Place the mirrors M_1 and M_2 close together leaving a space of about 1/3 mm. The reflecting surfaces must (1) be made parallel (for the Case III) or nearly so (for Cases I and II) and (2) be brought to the same level. Focus the telescope on the slit or filament. Make the reflections take place at the crack M_1M_2. The mirrors will be parallel when the two images of the source exactly superpose, provided the two mirrors are at the same level. One, of course, brings the two mirrors to nearly the same level by eye. This can be done quite accurately since they are very close together. The exact adjustment is indicated (after slow variations in level about the approximate setting) by the appearance of the fringes.

When fringes appear, they will probably be fuzzy and skewed. Further careful adjustment of the source-slit or line filament and of the relative angular settings of the mirrors M_1 and M_2 will make the fringes sharp and straight.

Further adjustment of the eyepiece or telescope position may be necessary to bring the fringes into the center of the bright spot in the field of view. This bright background is now removed by screening off most of the mirror surfaces of M_1 and M_2, leaving uncovered only a narrow strip near the intervening crack. These strips are made so narrow that only the fringes are left in the field of view of the telescope. This completes the adjustment for Cases I and II.

Case III. Multiple reflection interferometer

The great beauty of this particular multiple reflection scheme is that its adjustment is the same, so far, as the adjustment for Case I. Two of the three mirrors are already adjusted relative to each other. If these mirrors are horizontal, two plumb lines serve to make the source-slit, the crack M_1M_2 and the telescope coplanar. The mirror M_3 is now brought into position and adjusted so that it will reflect the light beams many times along the crack M_1M_2. The coarse angular adjustments suffice for this. The separation, C, of M_3 from M_1 and M_2, together with the angle of incidence, determines the number of reflections used. The adjustment of this separation is again not critical. (It can be made with a setscrew clamp on a vertical rod.)

In order to avoid diffraction losses caused by many transmissions through a narrow slit, the diaphragms that screen off the undesired light in the eyepiece limit the last reflection only. This stop can either lie on the mirror surface or be clamped near it.

As a further refinement, one can make M_3 shorter than M_1 and M_2 by an amount equal to $2C \tan i$ where C is the separation of M_3 from M_1 and M_2. This of course lets the tallest possible set of fringes through the apparatus for chosen values of the angle of incidence and separation C. The values of C and i also fix the number of reflections attainable. (In the actual tryout this refinement was not used.)

The instrument, thus adjusted, is n times more sensitive than the orthodox types of interferometers using, as they often do, only a single reflection from a moving mirror. Since n may be 14 or even 20, the gain is quite worthwhile, especially since it is to be had without greatly increased difficulty in adjustment of the apparatus.

Case IV. Compound multiple reflections—two opposed fringe systems

The adjustments in this case are essentially as in Case III except that they must be made for two separate fringe systems. M_1 and M_2 must have accurately parallel faces. As in Case III these mirrors can be adjusted before M_3 and M_4 are mounted on the instrument. M_4 needs two angular adjustments and one vertical adjustment. These can be quite coarse as they also may be for M_3. The distances $\overline{M_4M_{1,2}}$ and $\overline{M_3M_{1,2}}$ (C' and C, respectively) need not be equal. The juxtaposition of the two fringe systems can be effected by a slight adjustment of M_3 or M_4.

This instrument is $2n$ times more sensitive than single reflection interferometers. If one obtains optical mirrors fulfilling the requirements on parallelism and thickness, this further increased sensitivity is again attained without much increased difficulty of adjustment.

Case IV has never been built on account of a lack of any imperative application for it.

The work which has been reported here was carried out at the Laboratories of the Gulf Research & Development Corporation. It is a pleasure to take this opportunity to thank Dr. Paul D. Foote, the Director of the Laboratory, for putting its excellent facilities at my disposal in the prosecution of these studies and for permission to publish them.

Multiple Sets of Fringes in the Michelson Interferometer

I. WALERSTEIN AND R. A. WOODSON, *Purdue University*

(Received January 27, 1936)

By using an unsilvered dividing plate in a Michelson interferometer or an intense source with the usual half-silvered plate several groups of interfering rays are produced due to internal reflections in the dividing and compensating plates. These lead to four distinct sets of interference fringes whose shapes change with variations of the distances and angles between plates and mirrors. They are circles, ellipses, straight lines and double sets of hyperbolas.

THE form and position of the fringes in a Michelson interferometer are dependent on the angle between the mirrors and the position of the mirrors. Their form is usually that of a conic section as was shown by Michelson[1] and later more rigorously by Krause.[2] The case where the two mirrors are exactly perpendicular to each other so that the film of air producing the path difference for interference has plane parallel sides was not treated by Michelson any further than to state that the fringes would be formed at infinity and would in general be circles. The form of these fringes was investigated by Waetzmann and Lummer[3] for the special case when the compensating plate is removed and by Krause[2] for the general case with a compensating plate. In none of these cases was the effect of multiple reflections in the plates, which becomes important with intense sources, discussed.

The use of a half-silvered dividing plate in a Michelson interferometer limits the formation of fringes mainly to the two beams which divide and reunite at a on the half-silvered surface as in Fig. 1. These fringes are the ones that are usually described and will be designated here as the α-type fringes.

When the planes of the two mirrors M_1 and M_2 are made exactly perpendicular to each other and an intense source of light is used several other sets of fringes can be observed at the same time. These owe their origin to pairs of interfering beams which arise from multiple reflections occurring in the dividing plate D or in the compensating plate P. To study the characteristics of these fringes it is best to employ a completely clear dividing plate in order to increase their distinctness. One set of fringes to be designated as the β-type is due to the division of the original beam from the source taking place at b on the first surface of D as in Fig. 1. The application of Fresnel's equations for the intensity of the emergent beams will show that the β-fringes are slightly less distinct than the α fringes for the same setting of D and P. Two other sets of fringes γ-type and δ-type are seen in Figs. 6 and 7 to arise from reflections at the side mirror

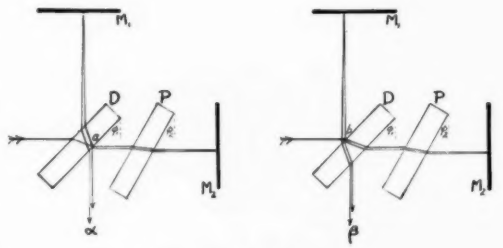


FIG. 1. Formation of α and β-fringes.

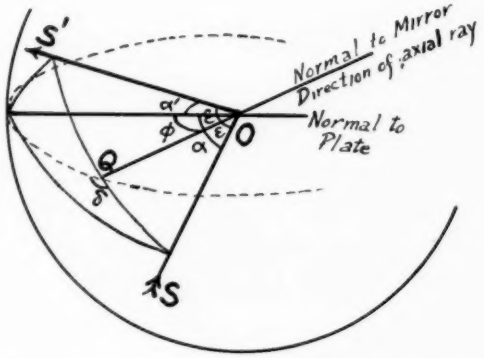


FIG. 2. Angles made by incident and reflected rays with plate and mirror.

[1] A. A. Michelson, Phil. Mag. **13**, 236 (1882).
[2] G. Krause, Ann. d. Physik **48**, 1037 (1916).
[3] E. Waetzmann and O. Lummer, Ann. d. Physik **36**, 383 (1911).

M_2 only. Their visibility is low for ordinary settings but improves as the angle between P and M_2 is increased from 45° towards 90°. Moreover these fringes can be viewed with M_1 completely screened off, and so their form and size can easily be followed against a black background. Even against the light background of the reflection from the mirror M_1 they can often be seen as in Figs. 3 and 5b.

To deduce the equation for these different sets of fringes when the plane of M_1 is truly perpendicular to the plane of M_2 we will consider that each of these is normal to the plane of Fig. 1 and denote the angles between D and M_2 as φ_1 and between P and M_2 as φ_2. A ray traveling in the plane of the paper and normal to M_2 will be designated as the axial ray. Consider a general ray not in the horizontal plane making an angle ϵ with the axial ray. It is incident on one of the plates at an angle α and after reflection from the mirror it strikes the same plate at an angle α'. In Fig. 2 SO and OS' represent the directions of this ray before and after reflection from the mirror. The plane SOS' cuts the plane of Fig. 1 (dotted circle) at an angle δ. The spherical angles α and α' can be expressed in terms of ϵ, δ and φ by the relations

$$\begin{aligned} \cos\alpha &= \cos\epsilon\cos\varphi+\sin\epsilon\sin\varphi\cos\delta, \\ \cos\alpha' &= \cos\epsilon\cos\varphi-\sin\epsilon\sin\varphi\cos\delta. \end{aligned} \tag{1}$$

If now a ray of light is incident at an angle α on a plate of thickness d and index of refraction n, and is divided into two rays one traveling through the glass and the other through a similar path in air, a path difference is introduced of an amount

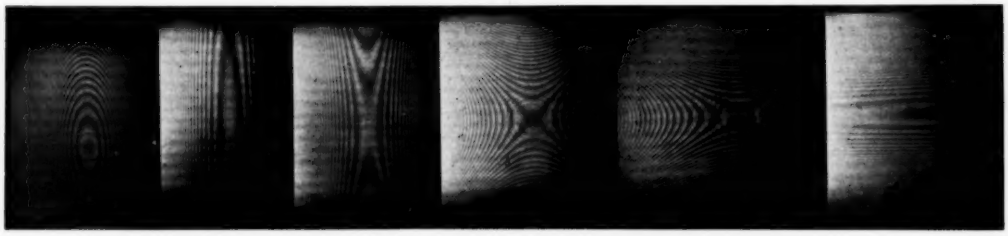
$$\Delta = d(n^2-\sin^2\alpha)^{\frac{1}{2}}-d\cos\alpha. \tag{2}$$

Similarly for light striking a plate after reflection from a mirror the path difference is

$$d(n^2-\sin^2\alpha')^{\frac{1}{2}}-d\cos\alpha'. \tag{2'}$$

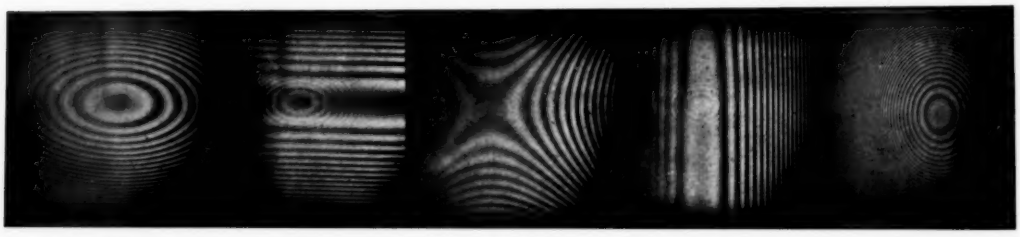
β-Fringes

In Fig. 1 let the difference in the perpendicular distance from b to M_1 and M_2 be L and let P have the same thickness and refractive index as D. A general ray is incident on D at an angle α_1,



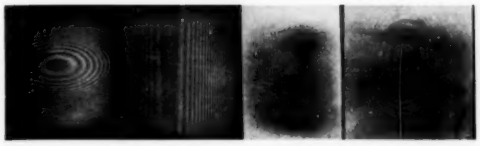
$L=6.5$ $L=5.65$ $L=5$ $L=3.61$ $L=2.5$ $L=1.57$

FIG. 3. $\varphi_1=\varphi_2=45°$. α-fringes are circles throughout increasing in diameter as L is decreased. Change of β-fringes from vertical ellipses to horizontal lines is indicated. For $L<1.57$ the β-fringes are horizontal ellipses. γ-fringes are seen as broad horizontal lines.



$L=1.62$ $L=1.16$ $L=0.56$ $L=-0.04$ $L=-2.02$

FIG. 4. $\varphi_1=45°$, $\varphi_2=30°$. β-fringes are ellipses throughout. For $L=1.63$ α-fringes have same eccentricity as β-fringes with separations larger than the β-ellipses. As L is decreased the α-fringes change through straight line and hyperbola stages to vertical ellipses. At $L=-2.02$ the eccentricities of the α vertical ellipses and β horizontal ellipses is the same.



a b c d

FIG. 5. (a) $\varphi_1=45°$, $\varphi_2=0$, $L=2.79$. The α and β-fringes are identical ellipses. The mirror M_2 was slightly tilted thus separating the centers of the ellipses and producing a hazy arc across the plate where the two ellipses don't quite match. (b) $\varphi_1=45°$, $\varphi_2=62°$, $L=3.7$. The condition is nearly that for which α-fringes are vertical lines and β-fringes horizontal lines. The γ-fringes can here be seen as sections of ellipses with center displaced to the left. The vertical shift of the center is due to P being tilted with respect to D. (c) $\varphi_1=45°$, $\varphi_2=75°$, $L=0.32$ mm. The α and β-fringes are hyperbolas with asymptotes inclined at the same angle to the y and x axis, respectively. (d) $\varphi_1=45°$, $\varphi_2=53°$. The mirror M_1 was blocked off and the γ-fringes are seen as vertical ellipses with center in the line of sight.

and on P at an angle α_2. Then from Eqs. (2) and (2′) it follows that

$$\Delta=d[(n^2-\sin^2\alpha_1)^{\frac{1}{2}}+(n^2-\sin^2\alpha_1')^{\frac{1}{2}}+(n^2-\sin^2\alpha_2)^{\frac{1}{2}}+(n^2-\sin^2\alpha_2')^{\frac{1}{2}}-(\cos\alpha_1+\cos\alpha_1')-(\cos\alpha_2+\cos\alpha_2')]-2L\cos\epsilon. \quad (3)$$

Expanding $\sin\epsilon$ and $\cos\epsilon$ in powers of ϵ up to the second, since ϵ is limited by the eye, we obtain from Eqs. (1) and (3)

$$\Delta=[d(n^2-\sin^2\varphi_1)^{\frac{1}{2}}+d(n^2-\sin^2\varphi_2)^{\frac{1}{2}}-2(c_1+c_2)-2L]-\epsilon^2[C_1+C_2-(S_1+S_2)\cos^2\delta-(c_1+c_2)-L], \quad (4)$$

where

$$c=d\cos\varphi, \quad s=d\sin^2\varphi/(n^2-\sin^2\varphi)^{\frac{1}{2}},$$
$$C=d\cos^2\varphi/(n^2-\sin^2\varphi)^{\frac{1}{2}}$$

and

$$S=d(n^2-1)\sin^2\varphi/(n^2-\sin^2\varphi)^{\frac{1}{2}}.$$

For the axial ray $\epsilon=0$ and Δ is the part in the first square bracket in Eq. (4) which shall be denoted by Δ_0. If the general ray produces part of a fringe which is in phase with the central fringe then $\Delta_0-\Delta=m\lambda$ where m is an integer. It follows that the expression in polar coordinates for the locus of pairs of rays which have a constant phase difference is

$$\epsilon^2[C_1+C_2-c_1-c_2-L]-\epsilon^2\cos^2\delta[S_1+S_2]=m\lambda.$$

By projecting these fringes on a plane tangential to the sphere at Q then the coordinates of S are $x=\epsilon\cos\delta$, $y=\epsilon\sin\delta$ for small values of ϵ. The equation for the β-fringes then becomes

$$[C_1+C_2-S_1-S_2-c_1-c_2-L]x^2+[C_1+C_2-c_1-c_2-L]y^2=m\lambda.$$

This represents a conic section. C, S, and c are positive for all values of φ_1 and φ_2 from 0° to 90°. The fringes can therefore never be circles. They are however horizontal or vertical ellipses, straight lines or hyperbolas depending on the setting.

α-FRINGES

A discussion of the ordinary α-type fringes observed in the Michelson interferometer is given by Krause, but it can be seen that a treatment similar to that for β-fringes will yield the following equation for the α-type.

$$[-C_1+C_2+S_1-S_2+c_1-c_2-L']x^2+[-C_1+C_2+c_1-c_2-L']y^2=m\lambda,$$

where L' is the difference in the perpendicular distances from a to M_1 and M_2. In our apparatus φ_1 was held at 45° and using the value of $d=7.58$ mm and $n=1.518$ it follows that $L'=L+10.73$. Substituting the values of C, S and c for $\varphi_1=45°$ the α and β-fringes become, respectively

$$[4.58+C_2-S_2-c_2-L]x^2+[2.54+C_2-c_2-L]y^2=m\lambda,$$

$$[6.15+C_2-S_2-c_2-L]x^2+[8.19+C_2-c_2-L]y^2=m\lambda.$$

For $\varphi_2=45°$ the α-fringes are circles. For all other angles they may be ellipses, straight lines or hyperbolas depending on the value of L. The

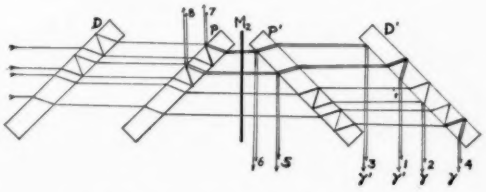


FIG. 6. Formation of γ and γ'-fringes.

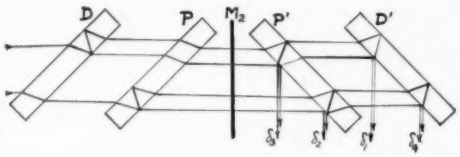


FIG. 7. Formation of δ-fringes.

hyperbolas are a double set which appear simultaneously in the field of view. Thus for $4.58+C_2-S_2-c_2>L>2.54+C_2-c_2$ and m positive the equation represents a set of hyperbolas which intersect the x axis, while for m negative they represent a second set which intersect the y axis. The two sets have common asymptotes—the asymptotic fringes being dark or light according as the center of the fringe system is dark or light. For various values of φ_2 the values of L can be calculated which will make the α or β fringes straight lines equilateral hyperbolas, etc. These positions were verified experimentally to within 0.02 mm or less in each case by using a mercury arc with a filter for the green line. Some of these were photographed and a few special cases of interest are shown and described in Figs. 3, 4 and 5.

The simultaneous appearance of the α and β fringes occurs only if the angle between M_1 and M_2 is very close to 90°. The centers of the two systems of fringes coincide when the angle is exactly 90°. By making one of the fringe systems elongated ellipses the motion of their center with respect to the center of the other fringes can easily be followed and can be utilized as a sensitive adjustment for the mirrors or for fine measurements of angle.

γ-Fringes

The γ-type of fringes are formed without the use of M_1. In Fig. 6 P' and D' represent the mirror images of P and D in M_2, and the rays reflected by M_2 are shown traveling to the right of M_2. The pairs of rays marked 2 and 4 have identical path differences whose value is

$$\Delta=\frac{2nd}{\cos r_1'}-2d\tan r_1'\sin\alpha_1'-\frac{2nd}{\cos r_2'}-2d\tan r_2'\sin\alpha_2'$$

$$=2d[(n^2-\sin^2\alpha_1')^{\frac{1}{2}}-(n^2-\sin^2\alpha_2')^{\frac{1}{2}}].$$

This leads to the equation of the fringes

$$(C_1-C_2-S_1+S_2)x^2+(s_1-s_2)x+(C_1-C_2)y^2=m\lambda.$$

This represents a conic section whose center is displaced to the left of the line of sight. From the expression for s_1 and s_2 it follows that the center is in the line of sight for $s_1=s_2$, i.e., $\varphi_2=45°$ or $\varphi_2=53°$. The solution $\varphi_2=45°$ is inadmissible here as the expression $(s_1-s_2)/2(C_1-C_2-S_1+S_2)=0/0$ leads to a determinate value of 0.07 rad. The experimentally determined value was $53\frac{1}{2}°$ and the fringes in that position are shown in Fig. 5d.

γ'-Fringes

The fringes formed by the pairs of rays shown at 1 and 3 as well as at 5, 6, 7 and 8 are identical in form with the γ-fringes but have their centers displaced to the right the same amount as the fringes are displaced to the left. They are weaker than the γ-fringes but they can be seen by either tilting P or D slightly so that the center of one system is displaced above or below that of the other. The fringes are vertical ellipses for all values of φ, their eccentricity increasing slowly with angle.

The γ-fringes can often be seen even against the background of the light from M_1 and can be observed as broad horizontal lines which are sections of ellipses in Fig. 3, and as ellipses in Figs. 5b and 5d.

δ-Fringes

A weak set of fringes best seen through a pinhole is produced by the light which passes through D or P and after reflection from M_2 passes through D or P again. Fig. 7 drawn similarly to Fig. 6 shows how these fringes are produced. It also indicates that they are visible in either D or P. Those fringes formed by reflections of rays in D and D' or by reflections in P and P' are identical in form. Denoting them by δ_1 and δ_2, respectively, their equations are

$$2(C_1-S_1)x^2+2C_1y^2=m\lambda$$

and

$$2(C_2-S_2)x^2+2C_2y^2=m\lambda.$$

They are circles for $\varphi=0$, changing into horizontal ellipses as φ grows and becoming horizontal straight lines at $\varphi=49°\ 4'$. At $\varphi=57°\ 15'$ the fringes become equilateral hyperbolas and finally at 90° vertical straight lines. All these forms were observed and at the proper angles. Those rays which are reflected by D and P' and by P and D' give rise to the δ_3 and δ_4-fringes represented by the equation

$(C_1+C_2-S_1-S_2)x^2 \mp (s_1-s_2)x+(C_1+C_2)y^2=m\lambda$. They also have their forms identical but their centers are displaced to the right and to the left of the line of sight, respectively. The displacement is zero when $s_1=s_2$ one solution of which is $\varphi=45°$.

If D is held fixed at $\varphi_1=45°$ and φ_2 is varied the δ_3 and δ_4-fringes become equilateral hyperbolas at $\varphi_2=70°$, and horizontal lines when $C_1-S_1+C_2-S_2=0$. This leads to a 4th degree equation in $\sin^2\varphi$. A solution can be obtained more easily by considering the straight lines as very long ellipses with center in the line of sight. This requires $s_1=s_2$ and choosing the second solution which was found for the γ-fringes we have $\varphi_2=53°$.

The δ_1 and δ_2-fringes are similar to those found by Lummer[4] in a modified Jamin interferometer but the δ_3 and δ_4-types have no counterpart in those experiments.

[4] O. Lummer, Ann. d. Physik **24**, 417 (1885).

JUNE, 1936 J. O. S. A. VOLUME 26

Intensity, Area, and Distance of Visual Stimulus. A Correction

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(Received April 30, 1936)

I REGRET extremely that an error was made in the calculation of equations in my article "Intensity, Area, and Distance of Visual Stimulus."[1] Herewith are the rectified equations which should be substituted for the incorrect ones on page 406 in the above, beginning with the following.

For the *small constellation*, this results in $\log k=1.60353$ and $n=-0.4803$. Then substituting these values in (1), we get

$$\log A=1.60353-0.4803 \log I \qquad (2)$$

which expresses the manner in which area of stimulus varies with intensity, at 30-cm stimulus-distance.

[1] Ellis Freeman, J. O. S. A. **22**, 402–407 (1932).

For the *large constellation*, following the same procedure, $\log k=0.66079$ and $n=-0.6272$. Then substituting these values in (1), we get

$$\log A=0.66079-0.6272 \log I \qquad (3)$$

which expresses the manner in which area of stimulus varies with intensity, at 300-cm stimulus-distance.

These rectifications in turn produce a change in the final parametric equation, which now becomes

$$\log A=1.70823-0.00349D -(0.4640+0.000544D) \log I. \qquad (4)$$

Aside from these modifications indicated above, all else in the paper remains unaffected.

The Effect of Length of Exposure of the Test Object on Visual Acuity. A Correction

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(Received April 27, 1936)

IN the issue of this journal for September, 1935, J. P. Conner and R. E. Ganoung[1] have made a statement which was based on an incorrect interpretation of experimental data published by us. In the belief that this statement should be corrected, we are submitting the present note.

Conner and Ganoung say: "According to Ferree and Rand, the effect of increases in exposure time on visual acuity is almost negligible with exposure times greater than 0.10 second for a background luminosity 5 lumens per square foot. C. E. Ferree and Gertrude Rand. Trans. I.E.S. **23**, 521 (1928)."

The study referred to did not treat of the effect of length of exposure of the test object on visual acuity. It pertained to the rise to its maximum or the development of the sensation difference between test object and background under certain experimental conditions. These conditions included the dying away of the positive afterimage of both white and black preexposures superimposed, respectively, on a black test object on a white background and a white test object on a black background. The study was made for the purpose of analyzing the difference we had found in speed of discrimination of black on white and white on black. The observations were made at two levels of intensity, 5 and 50 footcandles, by determining in serial order the lengths of exposure required to change in just noticeable steps the sensation difference between test object and background from threshold to maximal value. The judgment was, then, in terms of difference in sensation between test object and background, not in terms of minimal space values as would be the case in making determinations of visual acuity. In fact, the test objects used (the broken circle) were purposely made larger than would ordinarily be used for determinations of acuity. The entire test object subtended an angle at the eye of 15 minutes of arc; the opening of the circle, 3 minutes.

We have of course recognized that time of exposure is, within limits, an important factor in the results obtained in determinations of visual acuity, and have published a great deal of data in which short exposures were used in the discrimination of detail. However, we have not ourselves determined either the optimum length of exposure for the discrimination of detail or the effect of exposures longer than the optimum; and so far as we are aware, no one else has investigated these points. The optimum length of exposure is no doubt subject to great individual variations, a most important factor in which is the speed of adjustment of the lens of the eye for clear seeing. For this and other reasons it follows that the more extreme variations in this time may be expected to occur between young and old eyes. The best one can do at the present state of our knowledge is to adopt the exposure time which is most suitable for the purpose of the problem and hold this time constant for all comparative work. Conner and Ganoung chose 3 seconds for their work. It may be noted in this connection that short exposures give sensitivity for detecting small differences in the effect of this or that condition or factor. Just what length of exposure should be chosen for a standard rating of visual acuity has not, so far as we know, been agreed upon.

[1] J. P. Conner and R. E. Ganoung, J. O. S. A. **25**, 288 (1935), (footnote 3).